Optimization Models in The Natural Gas Industry

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Abstract

With the surge of the global energy demand, natural gas plays an increasingly important role in the global energy market. To meet the demand, optimization techniques have been widely used in the natural gas industry, and yielded a lot of promising results. In this chapter, we give a detailed discussion of optimization models in the natural gas industry with the focus on the natural gas production, transportation, and market.

Key Words: natural gas industry, optimization, gas recovery, gas transmission, gas market, Mixed Integer Programming (MIP), Mixed Integer Nonlinear Programming (MINLP)

1 Introduction

Concerned about global warming and shortage of crude oil, people become more interested in natural gas which is a relatively clean energy source and abundant in many places. Natural gas mainly consists of methane, and when burnt, it releases a fair amount of energy and less green house gases (*e.g.*, CO_2) than oil and coal. As we can see from Fig. 1, the world gas consumption/production is linearly growing since 1980 from approximately 52,890 billion cubic feet to approximately 104,424 billion cubic feet in 2006, according to the US Department of Energy [45]. Moreover, the natural gas consumption is expected to continue to grow linearly to approximately 153 trillion cubic feet in 2030, which is an average growth rate of about 1.6 percent per year [3].

In 2008, the residential use of natural gas accounted for 21%, the commercial use for 13%, the industrial use for 34%, the transportation for 3% and the electric power production for 29% [2]. The industrial sector

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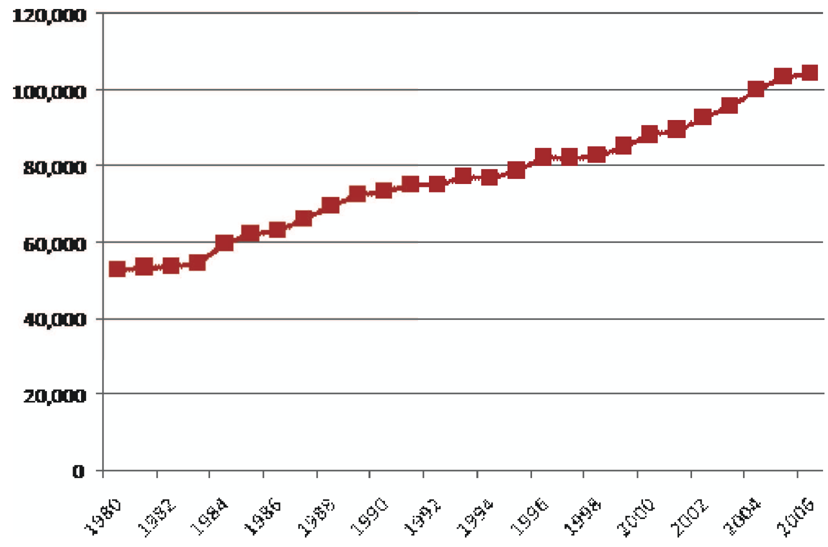

Figure 1: World gas consumption in billion cubic feet [45].

is expected to remain the largest end-use sector for natural gas though 2030 with an expected share of 40% [3]. The electric power generation from natural gas was the second largest consumer of natural gas after the industrial sector in 2006. The electricity generation accounted in 2006 for 32% of the world's total natural gas consumption. Due to the worldwide discussions/attempts to reduce green house gas emissions, the electricity generation via natural gas is expected to becoming even more important and its share of the world's total natural gas consumption is expected to increase to 35% in 2030 [3]. Hence, natural gas remains an important source of energy for both the industrial and the electricity sectors.

This chapter discusses different optimization models in the natural gas industry. We focus on three different key applications: the natural gas production, the natural gas transportation, and the natural gas market. This chapter is organized in such a way that we start with the introduction of the problem itself, and then discuss a mathematical formulation of the problem and finally review solution techniques to solve these models. However, especially when well known algorithms, such as Branch & Cut, are used to solve the mathematical programs, then we do not go into details but refer to the literature instead.

Section 2 discusses the optimization applications in gas recovery and production. Two problems are discussed in this section, the production scheduling problem and the maximal recovery problem. Section 3 focuses on gas transportation, where the network design problems and the optimal fuel cost problem are discussed. The natural gas market is discussed in Section 4, where both regulated and deregulated market models are considered. We conclude with Section 5.

2 Optimization in Gas Production (Recovery)

There is still a huge amount of gas natural gas reserves in the world: in 2009, the reserves were estimated at 6,254 trillion cubic feet; 69 trillion cubic feet above the estimate for 2008 [1]. This follows the general upwards trend of the world natural gas reserves over years. With a share of approximately 40.7%, the Middle East has the largest natural gas reserves of the world, followed by Eurasia with 32.2% and Africa with 7.8% [1]. On the country level, Russia has approximately 26.9% of the worlds natural gas reserves and holds together with Iran (15.9%) and Qatar (14.3%) approximately 57% of the world's natural gas reserves while the top 20 countries hold together 90.7% [1]. Interestingly, for most regions, the reserves-to-production rates are substantial, with an worldwide estimate of 63 years [11]. Hence, natural gas production and recovery will continue to be an important task in the future.

Optimization models and techniques are applied extensively in natural gas recovery processes, such as production scheduling, placement of well head, gas recovery systems or facilities designs. For a survey on gas and oil recovery and production, we refer the reader to Horne [26]. These optimization problems are computationally difficult to solve. One reason is that a huge number of parameters are subject to uncertainties. Another reason are the nonlinear/nonsmooth/nonconvex functions and constraints, due to the properties of gas production operations as explained in [9]. In the following, we discuss some specific optimization problems occurring in the gas production.

2.1 Production Scheduling Considering Well Placement

Usually, a gas reservoir is accessed by drilling multiple wells on its surface. Also gas withdrawal from any of the wells will lead to pressure reductions at all wells drilled on the same reservoir. Then the pressure reductions will come back to decrease the withdrawal rate at every well for the next period. The optimal production scheduling problem is to find the optimal withdrawal rate at every drilled well at each time period while determining the well location at the same time.

2.1.1 Mixed Integer Linear Programming Formulation

Murray and Edgar [34] formulate this problem as a mixed integer linear programming (MILP) problem. They try to determine the optimal well configuration (withdrawal rates) while satisfying the demand schedule without exceeding it. Drilling or not at a particular location, i, can be denoted by a binary variable, say, y_i. Hence, the drilling decision can only be made at particular locations i which have to be identified beforehand. Also, use q_i^k to denote the withdrawal rate from well i at time period k. The interaction between withdrawal rates and pressures at all the wells can be delineated by the following gas flow equation,

$$\nabla k_g \nabla \Phi + q = \phi c_t \frac{\partial \Phi}{\partial t}, \tag{1}$$

where $\Phi = 2\int_0^p \frac{\rho}{z(\rho)\mu(\rho)} d\rho$. Including this constraint in a mathematical programming formulation leads to huge computational difficulties. However, as stated in [34], this nonlinear constraint has a very good linearization substitute, called influence equations [5, 46]. In these equations, the pressure drop at well i is a linear function of withdrawal flow rates from all drilled wells. This is defined by influence function matrices, Φ^k, $k = 1, \dots, m$, where Φ_{ij} denotes the pressure drop at well i for a unit flow at well j during time period k. The maximal profit problem can be formulated as follows,

$$\max \quad \sum_{k=1}^{m} \sum_{i=1}^{n} b_i^k q_i^k \tag{2}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} \Phi_{ij}^k q_j^k = p_i^k, \qquad i = 1, \dots, n, k = 1, \dots, m, \tag{3}$$

$$\sum_{j=1}^{n} \Phi_{ij}^k q_j^k \leq \bar{p}_i^k, \qquad i = 1, \dots, n, k = 1, \dots, m, \tag{4}$$

$$\sum_{k=1}^{l} \sum_{j=1}^{n} \Phi_{ij}^k q_j^k \leq \hat{p}_i^l, \qquad i = 1, \dots, n, l = 1, \dots, m, \tag{5}$$

$$\sum_{j=1}^{n} q_j^k \leq d^k, \qquad k = 1, \dots, m, \tag{6}$$

$$q_i^k \leq M_i y_i, \qquad i = 1, \dots, n, \tag{7}$$

$$q_i^k \geq 0, \qquad i = 1, \dots, n, k = 1, \dots, m, \tag{8}$$

$$y_i \in \{0, 1\}, \qquad i = 1, \dots, n, \tag{9}$$

where, from well i during time period k, b_i^k is the benefit of one unit gas flow, p_i^k is the pressure reduction, and $\bar{p}_i^k$ is the maximal pressure reduction at period k. $\hat{p}_i^l$ is the maximal total pressure drop allowed from the initial time point to time period l and d^k is the demand at time k. M_i is a big number to bound the withdrawal flow rate if $y_i = 1$. Its objective function is the total benefit from the withdrawal of gas. Constraints (3) compute the pressure drop at each well location during every time period. Constraints (4) specify the upper bound by which the pressure can drop during a specific single period for each well location. Also there is an upper bound by which the pressure can drop during the period between the initial time point and the current time period, which is stated in constraints (5). Constraints (6) ensure that the total gas withdrawal from all wells does not exceed the demand at each time period. Constraints (7) show that only drilled wells can have a positive withdrawal flow rate. This results in a mixed integer programming (MIP) problem, which can be solved by well Branch & Bound or Branch & Cut techniques. We refer the reader to [27, 29, 36, 51, 28] for comprehensive discussions of these techniques.

Let us discuss now the drawbacks of the proposed model (2) - (9). The model does not include any other cost such as well drilling cost, it does not take into account the relationship between the profit coefficient

b_i^k and the demand d^k, and it assumes that the operator can choose any flow rate without considering the concurrent wellhead pressure. Also, after the deregulation of the natural gas market, the constraint (6) is not necessary and can be incorporated into the objective function instead. Furthermore, the different periods are intercorrelated to each other. For instance, the price of gas at time period t will affect the demand at the next time period $t+1$ and vice versa. By incorporating all these factors, a nonlinear mixed integer programming problem can be formulated.

2.1.2 Nonlinear Programming Formulation

A multiple-stage nonlinear optimization problem is also proposed by Murray and Edgar in [34]. They formulate a nonlinear problem for each time period taking into account the interactions between two consecutive stages. The objective function for each time period k incorporates more factors such as the well placement cost, compressor operating cost, compressor setup cost, and the price of gas, which is shown as follows,

$$f^k = \sum_{j=1}^{n} \left(A q_j^k - C_w \frac{q_j^k}{q_j^k + \epsilon} - U_j^k q_j^k - C_j^k q_j^k + D_j^k \right), \tag{10}$$

where A is the price per unit gas flow, and C_w is the setup cost of any well placement. Instead of using the binary variables y_i to denote whether a well is drill or not, this nonlinear programming formulation uses the term $\frac{q_j^k}{q_j^k+\epsilon}$ to approximate y_i, where ϵ is a small constant compared to the magnitude of gas withdrawal flow rates q_j^k, $j = 1, \ldots, n,$, $k = 1, \ldots, m$. To be able to use this approximation, the magnitude of the flow rates are assumed to be known. U_j^k is the operating cost of the compressors for a unit flow of q_j^k. $-C_j^k q_j^k$ and D_j^k approximate the setup cost of a compressor at this location before time period k. Setting $D_j^k = C_j^k q_j^{k-1}$ makes the summation of these two terms equal to 0, which ensures that the compressor setup cost only occur once. For the nonlinear formulation, the deliverability equations are considered besides the constraints in the MIP formulation. The deliverability constraints specify the relationship between the withdrawal rate and well head pressure, which is also approximated by linear functions and shown as follows,

$$q_j^k \leq e_j^1 + e_j^2 \rho_j^k, \qquad j = 1, \ldots, n, k = 1, \ldots, m, \tag{11}$$

where e_j^1 and e_j^2 are the linear coefficients and ρ_j^k is the bottom-hole pressure at well site j after time period k.

Also a multi-stage based algorithm is proposed in [34], in which all stages (time periods) are solved in an sequential order from 1 to m. We describe this algorithm as follows:

Step 1: Set up the problem: obtain parameters, e_j^1 and e_j^2, by some regression techniques; assume that no compressor is needed initially and set $U_j^1 = C_j^k = D_j^k = 0$; start from the first period problem.

Step 2: Solve the period k problem with an appropriate nonlinear programming algorithm, such as the gradient projection method [42].

Step 3: Examine the dual variables of the deliverability constraints. If none is positive, an optimal solution has been found for time period k, then go to Step 6. Otherwise, go to Step 4.

Step 4: If all positive dual variables are associated with deliverability constraints of the lowest feasible delivery pressure, an optimal solution is found for time period k, then go to Step 6. Otherwise, go to Step 5.

Step 5: Select the deliverability constraint with the largest associated dual variable, and then relax this constraint to the next lowest delivery pressure. Go to Step 2.

Step 6: By using the current period optimal solution, update the parameters in the next period problem. If $k = m$, terminate the whole program. Otherwise, set $k = k + 1$, and go to Step 2.

The drawback of the proposed model is that it does not consider all time periods together but considers them separately. Obviously, with this approach, an optimal solution to the practical problem cannot be obtained, as the interactions among all time periods are not taken into account.

2.2 Total Gas Recovery Maximization - An Optimal Control Formulation

In order to withdraw as much natural gas from a reservoir as possible, one option is to use waterflooding. This leads to the following immediate question. What is an optimal water injection rate with respect to different objectives, such as the maximal ultimate recovery, or the total revenues? A lot of models have been proposed for this problem. Mantini and Beyer [31] proposed optimal control models to this system and defined several objective functions due to different aspects of the problem.

Now, suppose there are two wells drilled on the surface of the gas reservoir, one for gas recovery and one for water injection. Therefore, let $r(t)$ denote the withdrawal rate of gas which is bounded by the maximum rate of gas extraction $r_m(t)$. Through the water injection, well water is injected into the reservoir at the nonnegative rate $s(t)$. This model assumes a constant g which is the ratio of gas entrapped behind the injected water to the volume of water at any time. The model to maximize the ultimate gas recovery can then be stated as

$$\begin{aligned}
\max \quad & \int_0^\infty r(t)dt & (12)\\
\text{s.t.} \quad & PV = NRT, & (13)\\
& \frac{dV}{dt} = -s(t) - gs(t), & (14)\\
& \frac{dN}{dt} = -r(t) - \frac{gs(t)P(t)}{RT}, & (15)\\
& r_m(t) \geq r(t) \geq 0, \\
& s(t) \geq 0,
\end{aligned}$$

where $P(t)$, $V(t)$ are the pressure and volume of the gas reservoir, $N(t)$ is the amount of gas which is not entrapped at time t. R is the universal constant of gas, and T is the temperature. Constraint (13) is the ideal gas law, constraint (14) shows the entrapped gas equals to constant g times the volume of the water while constraint (15) states that gas is entrapped at the current pressure in the reservoir and remains at the same pressure and has no effect on the reservoir. By introducing another variable $Q = P/RT$ and plugging constraint (13) into constraint (15), a more concise model can be obtained

$$\begin{aligned}
\max \quad & \int_0^\infty r(t)dt \\
\text{s.t.} \quad & \frac{dV}{dt} = -(1+g)s(t), \\
& \frac{dQ}{dt} = \frac{-r(t) + P(t)s(t)}{V(t)}, \\
& r_m(t) \geq r(t) \geq 0, \\
& s(t) \geq 0.
\end{aligned}$$

Mantini and Beyer [31] also discuss several other objective functions. For example, the objective function to maximize the present worth value of the net revenues for internal rate of return, ρ, not equal to 0, is

$$\beta \int_0^\infty e^{-\rho t}[r(t) - \alpha s(t)]dt,$$

where α is the ratio of the water price (per cubic meter) to the gas price (per mole), and β is the gas price per mole. Due to the presents of the differential equations, these problems are generally computationally difficult to solve. However, Mantini and Beyer established a very interesting theorem, characterizing the properties of (some) optimal solutions of the control variable $r(t)$ and $s(t)$. Let us re-state this theorem here.

Theorem 2.1. *[31] The objective function $\int_0^\infty r(t)dt$ is maximized by any functions $\hat{r}$ and $\hat{s}$ such that,*

$$\int_0^{t_1} \hat{r}(t) = V_0(P_0 - P_c), \tag{16}$$

$$\int_0^{t_2} \hat{r}(t) = \frac{P_c(V_0 - V_c)}{1+g}, \tag{17}$$

$$\hat{r}(t) = 0, \forall t > t_2, \tag{18}$$

$$\hat{s}(t) = \begin{cases} 0, & 0 \leq t < t_1, \\ \frac{\hat{r}(t)}{P_c}, & t_1 \leq t \leq t_2, \\ 0, & t > t_2. \end{cases} \tag{19}$$

for t_1 and t_2 are any numbers with $0 < t_1 < t_2$, where P_0 and V_0 are the initial pressure and volume respectively and gas recovery stops when $P \leq P_c$ or $V \leq V_c$.

This theorem leads to the interesting statement that it is optimal to start the waterflooding when the first time P is lower than P_c; that is, the entrapped gas is at the lowest possible pressure. Although, in practice, this may not be valid for some specific gas wells due to discrepancies between modeling and reality.

3 Natural Gas Pipeline Network Optimization

Originally natural gas was treated as a byproduct of crude oil or coal mining and was spared. The flares in the mining field were usually natural gas [44]. Not until the introduction of pipelines did the natural gas become one of the major sources of energy. The earliest gas pipelines were constructed in the 1890's and they were not as efficient as those that we are using nowadays. The modern gas pipelines did not come into being until the second quarter of twentieth century. Because of the properties of natural gas, pipelines were the only way to transport it from the production sites to the demanding places, before the concept of Liquefied Natural Gas (LNG). The transportation of natural gas via pipelines remains still very economical, but it is highly impractical across oceans. Although LNG market is burgeoning in high speed now, pipeline network remains the main transportation system for natural gas.

Gas pipelines play a major role in energy supply and security. The Nord Stream Gas Pipeline (NSGP) project, transporting Russian gas to Germany, is one of the recent large scale pipeline projects [15]. The NSGP is planned as a twin-pipeline with a total capacity of 55 billion cubic meters per annum. The estimated investment cost are 4 billion €, financed by a joint venture of the three companies JSC Gazprom, BASF AG and E.ON AG. Not least, the decision to build the marine pipeline was driven politically, passing by Poland, Lithuania, Estonia, Belarus and Ukraine, in order to increase the natural gas supply security for Germany, mainly.

After the post war gas pipeline boom, a lot of research has been done in optimization applications to pipeline networks; for instance, how to setup the pipeline network, how to determine the optimal diameter of the pipelines, how to allocate compressor stations in the pipeline network, and what is the minimal fuel consumption of the network. Typically, the mathematical programming formulations of the pipeline optimization problems contain a lot of nonlinear/nonconvex/nonsmooth constraints and functions. The most common constraints are the so-called Weymouth panhandle equations, which relate the pressure and flow rate through a segment of pipeline (i, j). They read as follows

$$\text{sign}(f_{ij}) f_{ij}^2 = p_i^2 - p_j^2, \qquad (i, j) \in A_p, \tag{20}$$

where f_{ij} is the flow rate of pipeline (i, j), p_i and p_j are the pressures at node i and j respectively. Hence, the direction of the gas flow depends on the pressure difference of the two nodes i and j. Therefore, the nonsmooth function $\text{sign}(f_{ij})$ is needed.

Recently, more research is related to the network optimization of gas transmission; given the network structure other than the design of the network topology. One of the few papers dealing with the design of network topology is the one by Rothfarb et al. [43], where the authors propose a tree generating algorithm to design the network topology.

3.1 Compressor Station Allocation Problem Considering Pipeline Configurations

Once a network topology is chosen, one problem is to determine the optimal configuration of the pipelines and the location of the compressor stations in this network. Because of the high setup cost and high maintenance cost, it is desirable to have the best network design with the lowest cost. This problem concerns a lot of variables: the number of compressor stations which is an integer variable, the pipeline length between two compressor stations, the diameters of the pipelines, and the suction and discharge gas pressures at compressor stations. This problem is computationally very challenging since it includes not only nonlinear functions in both objective and constraints but, in addition, also integer variables.

A simple and typical network for this type of problem is shown in Fig. 2. Node s is the supply node where the gas is produced. Nodes a and b are the demand nodes where the gas is consumed. The trapezoids 1 through 6 denote the compressor stations. There are three branches: s to 3 is the first branch, 3 to a is the second branch, and 3 to b is the third branch.

Suppose there are at most n compressor stations to be set up, and n_1, n_2, and n_3 denote the number of compressor stations on branch 1, 2, and 3 respectively. For each pipeline segment i, there are five associated parameters: the flow rate f_i, the discharge pressure (from the upstream compressor) p_i^d, the suction pressure (from the downstream compressor) p_i^s, the diameter d_i, and the length l_i.

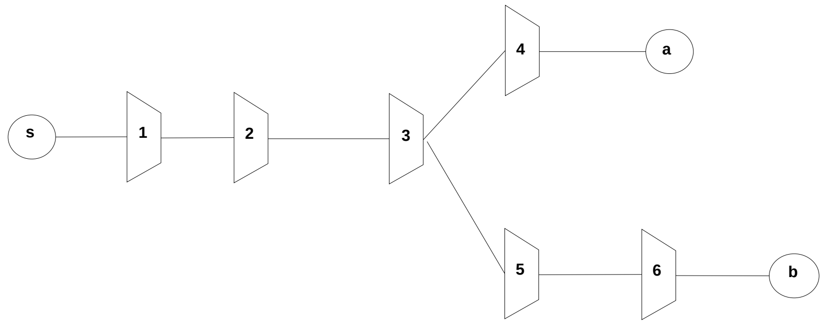

Figure 2: A gas pipeline network configuration problem with three branches.

The formulation for the three branches problem by Edgar et al. [19, 20] reads as follows

$$\min \quad \sum_{i=1}^{n}(O_y + C_c)\alpha\frac{T_s}{\eta_s}\frac{\gamma}{\gamma-1}\left[1-\left(\frac{p_i^d}{p_i^s}\right)^{\frac{z(\gamma-1)}{\gamma}}\right] + \sum_{i=1}^{n+1} C_l l_i d_i \tag{21}$$

$$\text{s.t.} \quad p_i^d \geq p_i^s, \qquad i = 1, \ldots, n, \tag{22}$$

$$p_i^d \leq K_i p_i^s, \qquad i = 1, \ldots, n, \tag{23}$$

$$\underline{p}_i^d \leq p_i^d \leq \bar{p}_i^d, \qquad i = 1, \ldots, n, \tag{24}$$

$$\underline{p}_i^s \leq p_i^s \leq \bar{p}_i^s, \qquad i = 1, \ldots, n, \tag{25}$$

$$\underline{l}_i \leq l_i \leq \bar{l}_i, \qquad i = 1, \ldots, n, \tag{26}$$

$$\underline{d}_i \leq p_i^d \leq \bar{p}_i^d, \qquad i = 1, \ldots, n, \tag{27}$$

$$f_i = A d_i^{\frac{8}{3}}\left[\frac{(p_i^d)^2 - (p_i^s)^2}{l_i}\right]^{\frac{1}{2}} \qquad i = 1, \ldots, n, \tag{28}$$

$$\sum_{i=1}^{n_1} l_i + \sum_{i=n_1+1}^{n_1+n_2} l_i = L_1, \tag{29}$$

$$\sum_{i=1}^{n_1} l_i + \sum_{i=n_1+1}^{n_1+n_3} l_i = L_2, \tag{30}$$

where γ is the ratio of specific heats, T_s is the suction temperature, z is the gas compressibility factor, η_s is the efficiency factor, O_y and C_c are cost functions with respect to horsepower. The objective function (21) contains two parts, of which the first is the compressor station costs and the second is the maintenance costs of the pipeline segments. Constraints (22)-(27) are the upper and lower bounds on pressures, pipeline lengths and diameters. L_1 and L_2 are the distances between the supply node and two demand nodes.

Model (21) - (30) can be solved by applying Branch and Bound techniques using reduced gradient nonlinear optimization method to solve the subproblem at each node in the Branch and Bound tree [19, 20]. Alternatively, a differential evolution method is proposed to solve this problem formulation by Babu et al.

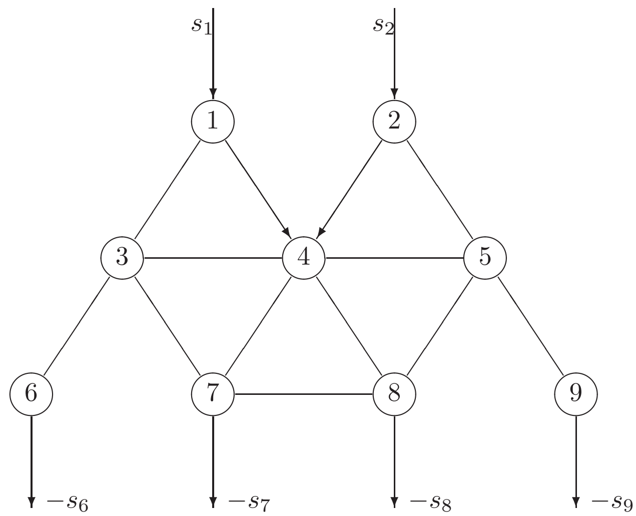

Figure 3: Least cost problem network.

[7]. The drawback of this model is that it highly depends on the topology of the network.

3.2 Least Gas Purchase Problem and Optimal Dimensioning of Gas Pipelines

In the modern natural gas industry, the gas production companies are rarely affiliated with the gas transmission and distribution companies. Thus, for gas distribution companies, one problem is to determine the best flow rate and gas pressures in each pipeline by which the least cost on purchasing gas from producers is achieved. This problem can be formulated as a optimization problem with linear objective function and nonlinear/noconvex constraints.

Consider now Fig. 3. s_1 and s_2 are the supplies for source nodes 1 and 2, the set of which is denoted by N_s. Nodes 6 to 9 are demand nodes with demands $-s_i$, $i = 6, 7, 8, 9$. In this model, there are two kinds of arcs: those with compressor stations such as $(1, 4)$ and $(2, 4)$, which is denoted by A_c; and those without, which are also called pipeline arcs and denoted by A_p. Flows on arcs with compressors are directed such that $f_{ij} \geq 0$, $\forall (i, j) \in A_c$, and flows on pipeline arcs are undirected and the direction depends on the pressures of both ends of this arc.

A mathematical programming formulation can be stated as

$$\min \quad \sum_{i \in N_s} c_i s_i \tag{31}$$

$$\text{s.t.} \quad \sum_{j \in A_i^+} f_{ij} - \sum_{j \in A_i^-} f_{ji} = s_i, \qquad \forall i \in N, \tag{32}$$

$$\text{sign}(f_{ij}) f_{ij}^2 = C_{ij}(p_i^2 - p_j^2), \qquad \forall (i,j) \in A_p, \tag{33}$$

$$f_{ij}^2 \geq C_{ij}(p_i^2 - p_j^2), \qquad \forall (i,j) \in A_c, \tag{34}$$

$$\underline{s}_i \leq s_i \leq \bar{s}_i, \qquad \forall i \in N, \tag{35}$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \qquad \forall i \in N, \tag{36}$$

$$f_{ij} \geq 0, \qquad \forall (i,j) \in A_c, \tag{37}$$

where p_i is the gas pressure at node i, c_i is the purchase cost per unit gas from supplier i, and C_{ij} an coefficient for arc (i,j), which is determined by the length, diameter and so on. A_i^+ denotes the set of arcs which are emanating from node i, while A_i^- denote the one of incoming arcs to node i.

The nonlinear constraints of the model above can be simplified by letting π_i substitute p_i^2. Then, constraints (33), (34), and (36) can be replaced by

$$\text{sign}(f_{ij}) f_{ij}^2 = C_{ij}(\pi_i - \pi_j), \forall (i,j) \in A_p,$$

$$f_{ij}^2 = C_{ij}(\pi_i - \pi_j), \forall (i,j) \in A_p,$$

$$\underline{\pi}_i \leq \pi_i \leq \bar{\pi}_i, \forall i \in N$$

With this substitution, the 'only' nonlinear functions left are $\text{sign}(f_{ij})$ and f_{ij}^2.

De Wolf and Smeers [50] propose a piecewise linear programming algorithm to solve this problem, in which they construct a piecewise linear approximation to the nonlinear constraints and solve the relaxed problem by simplex algorithm extensions [47]. The performance of the algorithm depends highly on the choice of the initial point. It is crucial to have a good starting solution, which can be obtained by solving the following problem:

$$\min \quad \sum_{(i,j) \in A} \frac{|f_{ij}| f_{ij}^2}{3C_{ij}^2} \tag{38}$$

$$\text{s.t.} \quad \sum_{j \in A_i^+} f_{ij} - \sum_{j \in A_i^-} f_{ji} = s_i, \qquad \forall i \in N,$$

$$\underline{s}_i \leq s_i \leq \bar{s}_i, \qquad \forall i \in N.$$

The objective function (38) in this problem is the amount of mechanical energy consumed in the gas pipeline per unit time. Its KKT necessary conditions (see [8]) is equivalent to the constraints (32), (33), and (35). The KKT necessary point is a good approximation starting point which does not take into account pressures' bounds and the existence of compressors. The algorithm proposed by [50] is as follows:

(o) **Initialization:** Let (f^0,p^0, s^0) be a vector of flows, pressures, and net supplies that satisfy constraints (32), (33), (34), (35), and (37). Replace the nonlinear function $\text{sign}(f_{ij})f_{ij}^2$ by a piecewise linear approximation including f_{ij}^0 as a breakpoint. Use f_{ij}^0 as starting point for the piecewise linear programming approach. Also set $k = 1$.

(i) **Iteration** k**:** Solve the approximation problem by the piecewise linear programming approach. Let (f^k,p^k,s^k) be the solution.

(ii) **Stopping rule:** Compute $\bar{f}_{ij}^k$ by the following equation:

$$\bar{f}_{ij}^k = \text{sign}(p_i^k - p_j^k)C_{ij}|(p_i^k)^2 - (p_j^k)^2|^{\frac{1}{2}}$$

If the error $e_{ij}^k = \bar{f}_{ij}^k - f_{ij}^k$ is greater than a given tolerance, for example, 10^{-5}, then add $\bar{f}_{ij}^k$ as a new discretization point and return to step (i). Otherwise stop and the incumbent solution is optimal.

It can be noticed that the optimal objective function value of problem (31) is a function of the diameters of the pipelines, say, $Q(D)$, because the parameter C_{ij} of pipeline (i, j) is a function of the diameter, where $C_{ij} = K_{ij}D_{ij}$ and K_{ij} is a coefficient. If the network structure and the length of each pipeline are fixed, the investment problem is to find the best pipeline diameters which achieve the lowest investment cost including both the gas purchase cost $Q(D)$ and the pipeline construction cost $C(D)$. They are given as

$$C(D) = \sum_{(i,j)\in A} = (k_G D_{ij}^2 + k'_G D_{ij} + k''_G)l_{ij},$$

where l_{ij} is the length of pipeline (i, j). Then the investment problem becomes

$$\begin{aligned} \min \quad & C(D) + Q(D) \\ \text{s.t.} \quad & D_{ij} \geq 0, \qquad \forall (i, j) \in A, \end{aligned} \tag{39}$$

which is a bilevel programming problem. The second part of the cost function, $Q(D)$, is nonconvex/nodifferential and has an implicit domain. De Wolf and Smeers [48] propose how to get one generalized subgradient, as in the next proposition.

Proposition 3.1. *Denote by f^*, s^*, π^* an optimal solution of the operations problem (31). Let w_{ij}^* be an optimal value of the dual variable associated to constraint (33). Then*

$$(\ldots, w_{ij}^* 5K_{ij}^2 D_{ij}^4, \ldots) \in \partial Q(D), \tag{40}$$

where $\partial Q(D)$ is the generalized subdifferential.

The investment problem (39) can be solved by a bundle method which performs well for nondifferential optimization problems. By using a bundle method, we do not need to know the explicit domain of the objective function. Hence it is a good fit for the investment problem because the objective function domain is

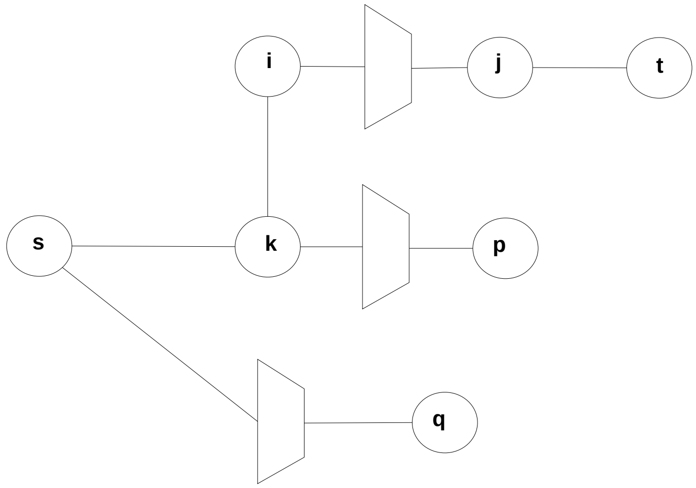

Figure 4: A gas pipeline network.

implicit. At each step, it only needs the value of the objective function and one of the generalized subgradient, which can be computed by (40). The dual variables, w_{ij}, can be obtained while solving the operations problems by using simplex algorithm extensions. Readers may find more comprehensive discussions of the bundle method in [25].

3.3 Minimum Fuel Consumption Problem

To let the consumer receive an acceptable withdrawal rate of gas, the pipeline needs to maintain a certain pressure. This is achieved by adding compressor stations in the network. One well known problem is the minimal fuel cost problem due to the fuel consumption of compressor stations, which are usually considered as special arcs in the network of this type of models. The minimal fuel cost problem has been widely discussed in the literature; see for instance [39, 40, 52, 41, 17, 24].

An typical gas pipeline network is shown in Fig. 4. Node s is the source node, and t, p, and q are the demand nodes. Arc (j, t) is an ordinary pipeline arc, arcs (i, j), (k, p), (s, q) are compressor station arcs. In each compressor station (i, j), there are C_{ij} compressors, and the pressures at i and j are denoted by p_i and p_j respectively. Let A' denote the set of compressor station arcs, A'' denote the set of ordinary pipe arcs, V

denote the node set. Then, the minimal fuel cost problem can be stated as

$$\min \sum_{(i,j)\in A'} g_{ij}(x_{ij}, p_i, p_j) = \sum_{(i,j)\in A'} \frac{\frac{x_{ij} Z_i R T_i}{\omega}[(\frac{p_j}{p_i})^{\omega} - 1]}{\mu_{ij}} \tag{41}$$

$$\text{s.t.} \quad \sum_{j\in A_i^+} x_{ij} - \sum_{j\in A_i^-} x_{ji} = b_j, \quad \forall i \in V \tag{42}$$

$$p_i^2 - p_j^2 = R_{ij} x_{ij}^2, \quad \forall (i,j) \in A'' \tag{43}$$

$$0 \le x_{ij} \le u_{ij}, \quad \forall i,j \in A \tag{44}$$

$$p_i^L \le p_i \le p_i^U, \quad \forall i \in V \tag{45}$$

$$(\frac{x_{ij}}{n_{ij}}, p_i, p_j) \in \mathcal{D}_{ij}, \quad \forall (i,j) \in A' \tag{46}$$

$$n_{ij} \in 0, 1, 2, \ldots, N_{ij}, \quad \forall (i,j) \in A' \tag{47}$$

where p_i^L and p_i^U are the lower and upper bounds on the pressure of node i. At each compressor station (i,j), u_{ij} is the capacity, $N_{i,j}$ is the total number of compressor, and x_{ij}, n_{ij} are the gas flow rate and number of compressor in use respectively. Also there are several other related parameters for (i,j): z_i is the gas compressibility factor, T_i is the gas temperature, μ_{ij} is the compressor adiabatic efficiency, and R_{ij} is a gas constant. The most complicated constraint is (46) in which $\mathcal{D}_{ij}$ is the feasible domain of compressor station (i,j) as for variable triplet $(\frac{x_{ij}}{n_{ij}}, p_i, p_j)$. The feasible domain is stated below by the set of equations,

$$\frac{h_{ij}}{s_{ij}^2} = A_H + B_H(\frac{q_{ij}}{s_{ij}}) + C_H(\frac{q_{ij}}{s_{ij}})^2 + D_H(\frac{q_{ij}}{s_{ij}})^3 \tag{48}$$

$$\mu_{ij} = \frac{C_E(\frac{q_{ij}}{s_{ij}})^2 + B_E(\frac{q_{ij}}{s_{ij}}) + A_E}{100} \tag{49}$$

$$S_{min} \le s_{ij} \le S_{max} \tag{50}$$

$$Surge \le \frac{q_{ij}}{s_{ij}} \le Stonewall \tag{51}$$

$$h_{ij} = \frac{Z_i R T_i}{\omega}[(\frac{p_j}{p_i})^{\omega} - 1] \tag{52}$$

$$q_{ij} = Z_i R T_i \frac{x_{ij}}{p_i n_{ij}} \tag{53}$$

In the above equations, q_{ij} denote the flow through the compressor unit, s_{ij} denote the speed of the compressor(s), and A_H, B_H, C_H, D_H, C_E, B_E, A_E are the compressor unit's constants.

This problem is very difficult to solve, and its solution algorithms are highly dependent on the topology of underlying network. Most of the algorithms for this problem are based on dynamic programming [40, 41, 39] and gradient search approaches [52]. Also meta heuristic approaches have been conducted, such as ant colony optimization [17] or genetic algorithms [24].

4 Natural Gas Market Models

Government regulation over the gas industry dates back to the early days of natural gas usage. At the first glance, this seams to be reasonable, as government and the public are the main users of natural gas and investments in the natural gas industry are tremendous. Not until the 1980s began the deregulation of this industry to improve both equity and efficiency of the natural gas market. Between the original producers and end users, there exists a variety of participants, each of which acts to optimize its own benefits. Under different government policies, a lot of natural gas market models are proposed. In this section we discuss optimization models of both a regulated and a deregulated gas market.

4.1 Reallocation Problem in a Regulated Natural Gas Market

O'Neil et al. [37] propose a model on how to allocate gas to users with different priorities under the government regulations when encountered a gas shortage emergency. In this model there are multiple gas transmission systems among which any two systems are not necessarily connected physically. All users are divide into 9 categories with priorities 1 through 9. The transportation network is composed of two types of arcs and nodes: the physical arcs and nodes which really exist in practice - denoted by A_{phy} and N_{phy}, respectively - and the pseudo counterparts which are for convenience of modeling - denoted by A_{pseudo} and N_{pseudo}, respectively. Let K_w be the set of users who withdraw gas from gas system w. This model also includes the panhandle constraints (20) for each of the pipeline arcs. However, instead of using the actual nonlinear constraints, this model incorporates two linearized approximation constraints in each iteration, which read as

$$-\epsilon_{ij} \le -f_{ij} + \alpha_i p_i - \alpha_j p_j \le \epsilon_{ij}, \quad \forall (i,j),$$
$$\epsilon_{ij}^1 \le p_i - p_j \le \epsilon_{ij}^2, \quad \forall (i,j),$$

where ϵ, ϵ_{ij}^1, and ϵ_{ij}^2 are parameters determined at each iteration through

$$\begin{aligned}
\epsilon_{ij} &= \alpha_1 |f_{ij}^{new}|, \\
\epsilon_{ij}^1 &= (1-\alpha_1)(p_i^{new} - p_j^{new}), \\
\epsilon_{ij}^2 &= (1+\alpha_1)(p_i^{new} - p_j^{new}), \\
\alpha_1 &= \max\{\alpha(\gamma_2)^m, \frac{1}{2}\delta_2\},
\end{aligned}$$

with the positive constants α, γ_2, δ_2.

The allocation algorithm proposed by O'Neil et al. [37] is as follows:

Step 0: Allocate the minimum amounts that all users must receive. If no feasible solution exists, then stop; no allocation exists under the specified parameters.

Step 1: Allocate gas according to the priorities within each transporter's system, starting with priority 1 and proceeding in ascending order of priority.

Step 2: Determine if priorities 1 through 5 are satisfied. If so, go to step 4. Otherwise, fix the lower (6 through 9) priority users, in pipelines with a shortage in any higher priority, at their lower bounds.

Step 3: Allocate gas according to the priorities within the entire system.

Step 4: Incorporate the linearized nonlinear constraints and find the optimal solution minimizing the amount transferred between systems, as in the optimization problem (54) - (64).

The linear programming formulation used in the allocation problem [37] can be stated as

$$\min \quad \sum_{(i,j)\in I} |f_{ij}| + \sum_{(i,j)\in S} f_{ij} \tag{54}$$

$$\text{s.t.} \quad \sum_{j\in A_i^+} f_{ij} - \sum_{j\in A_i^-} f_{ji} = s_i - \sum_{k\in K}\sum_{l=0}^{9} d_{ikl}, \qquad \forall i \in N, \tag{55}$$

$$\sum_{i\in N}\sum_{k\in K} d_{ikl} + u_l = \bar{d}_l, \qquad l = 0, \ldots, 9, \tag{56}$$

$$\sum_{l=1}^{5}\sum_{k\in K_w}\sum_{i\in N} d_{ikl} + r_w = g_w, \qquad \forall w \in W, \tag{57}$$

$$-\epsilon_{ij} \le -f_{ij} + \alpha_i p_i - \alpha_j p_j \le \epsilon_{ij}, \qquad \forall (i,j) \in A_{ps}, \tag{58}$$

$$\epsilon_{ij}^1 \le p_i - p_j \le \epsilon_{ij}^2, \qquad \forall (i,j) \in A_{vc}, \tag{59}$$

$$0 \le s_i \le \bar{s}_i, \qquad \forall i \in N, \tag{60}$$

$$\underline{d}_{ikl} \le d_{ikl} \le \bar{d}_{ikl}, \qquad \forall i \in N, k \in K, l = 0, \ldots, 9 \tag{61}$$

$$\underline{p}_i \le p_i \le \bar{p}_i, \qquad \forall i \in N, \tag{62}$$

$$u_l \ge 0, \qquad l = 0, 1, \ldots, 9, \tag{63}$$

$$r_w \ge 0, \qquad \forall w \in W, \tag{64}$$

where s is the supply, d is the demand, u is the slack variable for the demand of each priority, and r is the slack variable for the demand of priority 1 through 5. In constraints (58), $-f_{ij} + \alpha_i p_i - \alpha_j p_j$ is the linearized version of the panhandle equation, where α_i and α_j are the coefficients of the first order Taylor series expansion. A_{ps} and A_{vc} denote the pipeline arc set and the compressor arc set, respectively. The objective function is the amount of gas transferred between two systems, I is the set of physical arcs that connect two systems, and S is the set of pseudo arcs that realize swapping by allowing flow into redistribution node. This is one of earliest mathematical models describing the natural gas market under regulation.

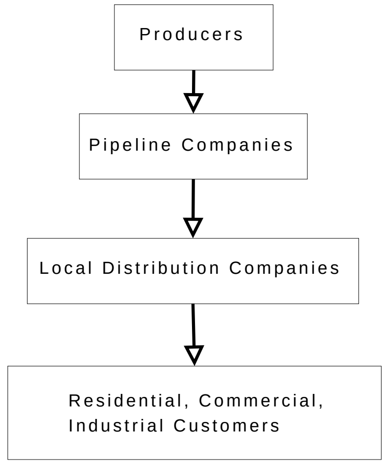

Figure 5: Participants Relationship in Regulated Gas Market.

4.2 Deregulated Natural Gas Market Models

In North America, before the 1980's, the natural gas market had been greatly regulated by the government since the 1930's. In the regulated market, there were primarily four participants: the gas producers, the gas pipeline companies, local gas distribution companies, and customers. The relationship of these participant is shown in Fig. 5, where producers sold gas to pipeline companies, and pipeline companies sold the gas to local gas distribution companies, and then local distribution companies sold the gas to various customers, such as industrial, commercial, and residential customers. In this regulated market, gas prices in each of the above transactions are tightly regulated by Federal and State governments as pipeline companies and local distribution companies had monopolies in the gas market. Since the mid 1980's, a series of deregulation policies have been announced. These polices encourage pipeline companies to switch from their traditional role as owners of natural gas by allowing producers and buyers to bypass the pipeline companies in that the buyers can transport their own gas through the pipeline system by paying some fees.

The deregulation of the gas market not only changed the roles of the former participants but also helped to create more participants, such as the gas marketing companies. Many models have been proposed for the deregulated gas market, especially for North America and Europe. Optimal purchasing strategies considering storage, contract, spot prices, peak day demands local distribution companies under North America gas market conditions have been studied by Avery et al. [6]. A model based on generalized network to provide optimal strategies for the marketing companies and local distribution companies, and a system, GRIDNET, to store all the dealed information were proposed by Brooks [14] and Brooks and Neill [13]. The Natural Gas Transmission and Distribution Module (NGTDM) is an important model of the North American gas market, which is a submodule of the U.S. Department of Energy's National Energy Modeling System (NEMS) and can be found in [4]. The Gas System Analysis Model (GSAM) is another North American gas market model, which tries to maximize the social welfare function to get the equilibrium, see for instance Gabriel et al. [23]. One of the most recent North American gas market models is the Mixed Complementarity-Based Equilibrium Model of Natural Gas Markets; see Gabriel et al. [22].

Gabriel et al. [22] consider six types of participants: the pipeline operators, the production operators, the marketers/shippers, the storage reservoir operators, the peak gas operators, and the customers. Each participant is trying to minimize cost or maximize profit for itself. For the sake of simplicity, this model assumes only linear relationship within each problem faced by a participant. Hence, every participant faces a linear programming problem. Because natural gas is a highly seasonal product, the model specifies three seasons in each year, which are denoted by $s = 1, 2, 3$. Every year has index $y \in Y$.

- $s = 1$: low demand season, Apr.-Oct.;
- $s = 2$: high demand season, Nov., Dec., Feb., Mar.;
- $s = 3$: peak demand season, Jan.

In this formulation, pipeline gas is available for all three seasons, and gas is injected to storage reservoir in season 1 and extracted in season 2 and 3, and peak gas is only used in the peak season.

The operator of pipeline a is trying to maximize its own profit by solving the following problem

$$\max \quad \sum_{y\in Y}\sum_{s=1}^{3} days_s \tau_{asy} f_{asy} \tag{65}$$

$$\text{s.t.} \quad f_{asy} \leq \bar{f}_a, \qquad \forall s, y, \tag{66}$$

$$f_{asy} \geq 0, \qquad \forall s, y, \tag{67}$$

where $days_s$ is the number of days in season s, τ_{asy} and f_{asy} are the prices and flow rates respectively of pipeline a in season s of year y. Constraints (66) are the upper bound constraints of the flows. τ_{asy} are the equilibrium show prices determined by the optimization problems of the other participants. Other than τ_{asy}, there are some other conditions relating this pipeline operator problem to the other pipelines and other kinds of participants. These conditions are usually called Market-Clearing conditions. The corresponding Market-Clearing conditions for the gas pipeline operator problem reads

$$days_1 f_{a1y} = \sum_{r\in R(n_1(a))} days_1 g_{ary} + \sum_{m\in M(n_1(a))} days_1 h_{am1y} \qquad \tau_{a1y} \text{ free} \quad \forall y \in Y, \tag{68}$$

$$days_s f_{asy} = \sum_{m\in M(n_s(a))} days_s h_{amsy} \qquad \tau_{asy} \text{ free} \quad s = 2, 3, \forall y \in Y. \tag{69}$$

These two Market-Clearing conditions state that all the supplies equal all the demands. g_{ary} is the flow rate of gas to storage operator r from the producers of season 1 through arc a, and h_{amsy} is the gas flow rate from producers of season s to marketer m through arc a.

The production operator's problem, for production company $c \in C$ at node $n \in N$, is to maximize its profit by solving the following problem

$$\max \quad \sum_{y \in Y} \sum_{s=1}^{3} days_s \big(\pi_{nsy} q_{csy} - c_c^{pr} q_{csy}\big) \tag{70}$$

$$\text{s.t.} \quad q_{csy} \leq \bar{q}_c, \qquad \forall s, y, \tag{71}$$

$$\sum_{y \in Y} \sum_{s=1}^{3} days_s q_{csy} \leq prod_c \tag{72}$$

$$q_{csy} \geq 0, \qquad \forall s, y, \tag{73}$$

where π_{nsy} and c_c^{pr} are the price of gas sold by the production company and cost to produce one unit of gas, respectively, for company c, and q_{csy} is the production rate of the company in season s of year y. Constraints (71) specify the upper bounds of the production rate in each period, and constraints (72) give the total production capacity for the whole planning horizon. Except this optimization problem, the coupling conditions for the production company c at node n are as follows,

$$\sum_{c \in C(n)} days_1 q_{c1y} = \sum_{a \in A_n^+} \Big(\sum_{r \in R(n_1(a))} days_1 g_{ary} + \sum_{m \in M(n_1(a))} days_1 h_{am1y} \Big), \quad \pi_{n1y} \text{ free} \quad \forall y \in Y, \tag{74}$$

$$\sum_{c \in C(n)} days_s q_{csy} = \sum_{a \in A_n^+} \sum_{m \in M(n_s(a))} days_s h_{asy}, \quad \pi_{asy} \text{ free} \quad s = 2, 3, \forall y \in Y. \tag{75}$$

The storage reservoir operator's problem, the marketer's problem, and the peak gas operator's problem are all described in the same way, first the linear programming problem and then the market-clearing conditions. Since all operator's problems are linear programming problems, the KKT conditions are necessary and sufficient. Combining all the KKT conditions and market-clearing conditions of every operator's problem, we then get a Linear Complementarity Problem (LCP), which is a special case of nonlinear complementarity problem (NCP) or variational inequality problem (VI). Gabriel et al. [22] proved that there exists a solution of the system and the prices are unique in this case. For more details about LCP, NCP, and VI, we refer the reader, for instance, to [18, 35, 21, 30].

Also a lot of models for the European gas markets have been proposed. A stochastic Stackelberg-Nash-Cournot equilibrium model for natural gas producers are proposed by De Wolf and Smeers [49]. Breton and Zaccour [12] propose a duopoly producer model. A recent European gas market model similar to the model in [22] is GASTALE proposed by Boots et al. [10].

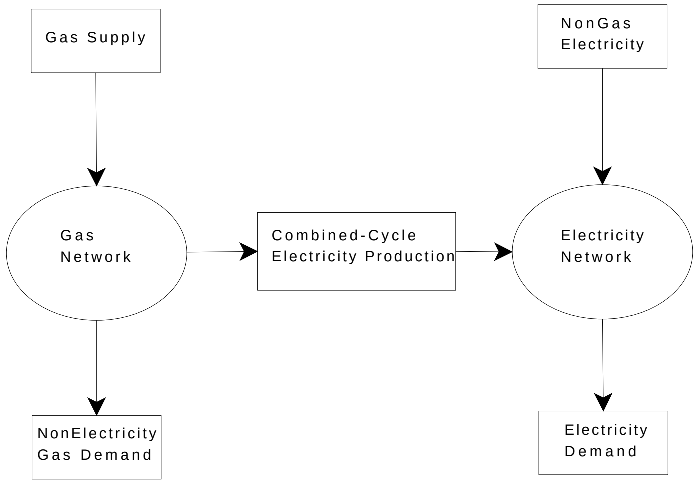

Figure 6: Relationship between Gas Network and Electricity Network.

4.3 Optimization in the Energy System Combining Natural Gas System and Electricity System

Natural gas is widely used in electricity production. Because combined-cycle plants are highly efficient and have less damage to the environment, more and more power plants of this type are build around the world. Hence the electricity and the gas system are now highly correlated. Here we discuss some related optimization applications regarding this relationship.

4.3.1 Electricity System Reliability Study using Natural Gas Transmission Network Modeling

Due to the increasing number of combined-cycle power plants being built, electricity production relies more and more on the amount of gas the power plants can get. However, the electricity plants are not the only users of natural gas; see Sec. 1. In order to perform a reliability analysis of the electricity system, it is important to study the maximal amount of gas which the gas network can supply to the electricity plants. The relation between gas network and electricity network is shown in Fig. 6. Munoz et al. [33] studied the problem of the maximal gas supply the electricity system can receive, taking into account the other gas users, the pipeline capacity and the production capacity. The formulation is very similar to the gas pipeline operations problem (31). Instead of minimizing the gas purchase cost as in (31), this problem maximizes the total electricity which can be produced by using gas from the gas system. It can be formulated as

$$\max \quad \sum_{i\in N_e} A_i e_i + B_i e_i^2 + C_i e_i^3 \tag{76}$$

$$\text{s.t.} \quad \sum_{j\in A_i^+} f_{ij} - \sum_{j\in A_i^-} f_{ji} = s_i - d_i - e_i, \qquad \forall i \in N, \tag{77}$$

$$\text{sign}(f_{ij}) f_{ij}^2 = C_{ij}(p_i^2 - p_j^2), \qquad \forall (i,j) \in A_p, \tag{78}$$

$$f_{ij}^2 \geq C_{ij}(p_i^2 - p_j^2), \qquad \forall (i,j) \in A_c, \tag{79}$$

$$\underline{s}_i \leq s_i \leq \bar{s}_i, \qquad \forall i \in N, \tag{80}$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \qquad \forall i \in N, \tag{81}$$

$$\underline{d}_i \leq d_i \leq \bar{d}_i, \qquad \forall i \in N, \tag{82}$$

$$\underline{e}_i \leq e_i \leq \bar{e}_i, \qquad \forall i \in N, \tag{83}$$

$$f_{ij} \geq 0, \qquad \forall (i,j) \in A_c, \tag{84}$$

where the objective function is a polynomial function of withdrawal of gas from the gas network. e_i is the gas withdrawal to produce electricity. d_i is the demand not related to electricity production. A_i^+ denotes the set of arcs which are emanating from node i, while A_i^- denotes the set of incoming arcs to node i.

Munoz et al. [33] solve the above problem in two phases. First, by dropping all nonlinear constraints, a mixed integer linear programming problem is obtained and then solved, where the integer variables denote the directions of flows in the pipeline segments. Second, by knowing the directions of flows from the phase I problem, a nonlinear problem is solved. However, two theoretical questions still remain in the correctness of optimality obtained by the method. First, it remains unanswered whether the solution from phase I will ensure the phase II problem to be feasible. Second, it is not true that the second phase problem is a convex problem for which a simple counterexample can easily be constructed, such as the single pipeline segment problem.

4.3.2 Optimization in Natural Gas Contracts

Many electricity production plants use a lot of sources among which natural gas is a very reliable alternative to meet the high electricity demand. The optimization of fuel contracts for a hydro-based power system is a very good example. In hydro power systems, precipitation varies from season to season. For the low precipitation seasons, the plants need to buy gas to generate electricity.

Let us now discuss a model which deals with the optimal dispatch strategy while considering the particular specifications of gas supply contracts as in Chabar et al. [16]. This model assumes a take-or-pay contract, which is widely adopted, especially in Europe. If a take-or-pay contract is signed, specifying a

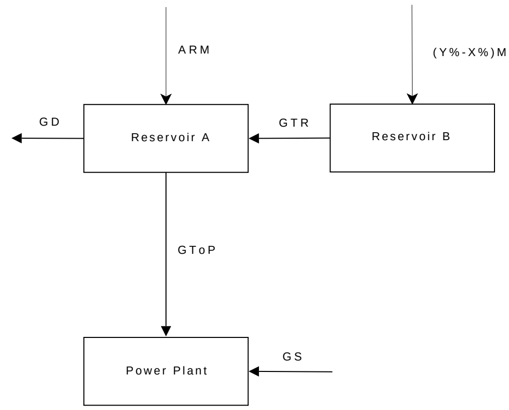

Figure 7: Gas Contracts Modeled by Reservoirs.

Table 1: Maintenance cycle length

Cycle	Frequency	Average Duration	Cost(MMR$)
Combustor	8000 hours	7 days	3.5
Hot path circuit	24000 hours	14 days	10
Major maintenance	48000 hours	21 days	20

monthly amount and a total annual amount, then at least $X\%$ of the monthly amount has to be bought every month and at least $Y\%$ of the contracted annual amount for the year has to be bought. Hence, there might be some gas excess based on contracts of this type. Two reservoirs are added into this model to accommodate the situations where gas excess exists. All excesses of gas not consumed monthly are stored in the gas reservoir A, the difference between the annual take-or-pay amount and the sum of all monthly take-or-pay amounts of the year is stored in reservoir B. Also, one of the gas contract provisions state that the gas purchased at any time point cannot "stay in the reservoir", or actually hold by the gas provider by more than N time periods, which means that if any amount of gas stays in the reservoir more than N time periods, it will have to be discarded. GD_t is used to denote the amount of gas discarded at time t. Figure 7 shows how the model, based on reservoirs, deals with the contract provisions.

Also the maintenance schedule is modeled by reservoirs. A fictitious remaining-hours reservoir is assigned to every power unit for each maintenance cycle. For a 3 power units 3 cycles problem, there will be 9 reservoirs. The length of each kind of cycle is shown in Table 1. For each power unit, the reservoirs are filled with the amount of remaining hours of operation until next maintenance. The capacity of each reservoir is the length of the cycle. As the unit operates, all reservoirs for that unit are reduced by the quantity of the elapsed hours. After maintenance, the fictitious maintenance reservoir is filled to its capacity.

Considering also the maintenance scheduling of the thermal plant, a dynamic programming formulation of the problem, for a given stage and price, is proposed by Chabar et al. [16]:

$$FBF_t^k(VA_t, VB_t, \{VH_t^{i,j}, i = 1, \dots, n, j = 1, \dots, m\}, \pi_t^k) \tag{85}$$

$$= \quad \max \quad RI_t + \sum_{s=1}^{S} p_{t+1}(k,s) FBF_{t+1}^s(VA_{t+1}, VB_{t+1}, \{VH_{t+1}^{ij}, i = 1, \dots, n, j = 1, \dots, m\}, \pi_{t+1}^k) \tag{86}$$

$$\text{s.t.} \quad VA_{t+1} = VA_t + ARM_t - GToP_t + GTR_t - GD_t, \tag{87}$$

$$VB_{t+1} = VB_t - GTR_t \tag{88}$$

$$VH_{t+1}^{i,j} = VH_t^{i,j}(1 - x_t^{i,j}) + \overline{VH}^j x_t^{i,j} - \gamma EG_t^i, \quad i = 1, \dots, n, j = 1, \dots, m, \tag{89}$$

$$\sum_{i=1}^{n} \psi_t^i EG_t^i = H_c(CToP_t + GToP_t + \nabla G_t), \tag{90}$$

where VA_t and VB_t are the volume of gas in reservoirs A and B, respectively, and $VH_t^{i,j}$ is the "volume" of remaining hours of operation that the unit i has until the next maintenance of cycle j. $GToP_t$ is the amount of gas actually used to generate electricity, and GS_t is the amount of gas purchased or sold to the gas spot market. ARM_t is the amount of gas purchased from the gas distributor, and should be bounded below by $X\%M$. GTR_t is the amount of gas transfer from A to B, and GD_t is the amount of gas discarded when it is in the reservoir more than the maximum storage time, N. n and m are the total number of power units and total number of maintenance cycles, respectively. RI_t is the immediate revenue in stage t. π_t^s is the spot price in stage t of scenario s. $p_{t+1}(k,s)$ is the transition probability of the spot price of scenario k in stage t to the spot price of scenario s in stage $t+1$. $x_t^{i,j}$ is the binary decision variable associated with the schedule of maintenance of cycle j for unit i at stage t. $\overline{VH}^j$ is the maximum capacity of the reservoir of remaining hours of operation until the next maintenance of cycle j. EG_t^j is the energy generated by unit i at stage t. γ is an inverse coefficient of the power unit, and ψ_t^i is conversion factor from MMBTU to MWh of unit i at stage t, and H_c is the heat rate of the gas.

Constraints (87)-(89) are the fictitious reservoir balance constraints and (90) is the transformation from gas to electricity. Except constraints (87)-(90), there are also a lot of other constraints, such as gas consumption priority constraints, maximum and minimum gas consumption constraints, maintenance constraints, constraints related to the mechanism implemented for the modeling of the contracts and so on. For this problem, each stage is a mixed integer linear programming problem. And the whole problem is solved by using stochastic dual dynamic programming, first proposed by Pereira and Pinto [38].

Also the natural gas market can be modeled as a natural gas value chain. The primary component is natural gas in this chains. Various market models are proposed and utilized in reality at different stages along this value chain, e.g., production, transportation and processing, storage, import terminals and markets, wholesale and retail markets. Please refer to the natural gas value chapter by Tomasgard et al., and [32] for more details about market models within the natural gas value chain.

5 Conclusion

This chapter discuss various optimization models occurring in the natural gas industry; focusing on three aspects: production, transportation, and market. As we can see, the natural gas industry is a complex system and in great need of optimization techniques to improve performance. Especially the nonlinear and nonconvex nature of the problems makes it computationally challenging to find good solutions. We observe that linearization techniques are a common method to tackle these nonconvex functions, often reducing the problem to a (series) of linear or mixed integer liner programming problems. With the computational power of computers increasing over the last decade, the use of meta-heuristics is become more and more popular; especially for problems which cannot be handled with the current MINLP solvers either due to the size of the problem or due to the degeneracy.

The deregulation of the gas market introduced additional modeling aspects and computational challenges: various (additional) stochastic elements have been added to the 'classical' problems. This underlying structure of the problems cannot be ignored by any serious model and we expect that future research will focus on stochastic models and, especially, on new techniques how to solve these (large-scale) practical problems when also integer and nonconvex, nonlinear functions are present.

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