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Critical States of Nuclear Power Plant Reactors and Bilinear Modeling

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Summary. We present a new system methodology for modeling of nonlinear processes in nuclear power plant cores. This methodology makes use of a variety of different approaches from different mathematical fields. The problem of modeling critical states is reduced to a bilinear subproblem. A scheme which provides stable parameter identification and adaptive control for the nuclear nuclear power plant described by the bilinear differential equation is presented. Abnormal events are found via a system-theoretical approach. Transitions to critical states can be detected by bilinear analysis of observed characteristics and by optimization of sensory measurements. Latent conditions and critical parameters in the reactor core are estimated trough a bilinear modeling.

Key words: Nuclear power plant; nuclear reactor; reactor core; critical states; bilinear modeling

1 Introduction

The use of nuclear power plants is very controversially discussed in our society. However, nuclear power is a sustainable energy source. It emits almost no greenhouse gases and, according to the 2006 annual report of the International Energy Agency [27], when it replaces coal-fired plants, the CO_2 emission can be reduced by about 6-7 million tonnes per year per 1 GW. After coal, the uranium fuels are the second most abundant sources of electric energy in the world, and it is, in contrast to oil, distributed among many countries. However, terrible accidents in the past, like in the Chernobyl power plant on April 26 in 1986, together with the enormous capital cost involved in nuclear power plants caused an end of the nuclear power boom in the 70th and 80th. As the politicians get more sensitive to climate change issues and the fears for the security of the supply of fossil fuels grows, the nuclear power

gets a new boost nowadays. An increase of the carbon prices to between \$10 and \$25/tCO_2 makes nuclear power economically competitive against coal or natural gas-based power generation; estimated by the International Energy Agency [27]. The International Atomic Energy Agency (IAEA) reported at the end of 2006 that 29 nuclear power plant reactors were under construction [28]. Just recently, The New York Times reported that the company "NRG Energy" applied to build two new nuclear power plant reactors in Texas, the first time in the United States after the "Three Mile Island" accident in 1979 [29]. This trend also includes other nations like Japan or England.

In 2006, about 16% percent of the global electricity was produced by nuclear power plants and at the end of 2006, 435 nuclear power plants were in operation in the world [28]. There are two mayor challenge for nuclear power plants: First, the waste management and second the safety of the reactors. Greenpeace provides a list of over 100 serious incidents in nuclear power plants since December 1952, reflecting only a small subset of all accidents [30]. All this together with the "revival" of the nuclear power, increases the demand for the save control of the nuclear power reactors.

One of the difficulties in safety engineering for nuclear power plant reactors is the problem of modeling and optimization [3, 7, 8, 14, 15, 18, 22, 26]. Very often, the mathematical models include highly nonlinear differential equations, for which design techniques are a complicated problem. Accurate models are known for physical processes, that can be accurately simulated together. However, the equations of motion consist of partial and ordinary differential equations coupled via their boundary conditions, a model that offers little to the control designer [11, 19, 21]. It is therefore a problem of considerable interest in developing explicit low-order models; once a design has been constructed using such a low-order model, it can be tested by comparing with a full high-order simulation [19, 13, 23].

An example is given by the nuclear power plant reactor. The channels of nuclear reactor cores, boilers and other chemical processes very often present problems embodying thermo-hydraulic systems. The dynamics of a channel system characterized by coupling between fuel pin and coolant flow in a reactor core may be represented by either linear or bilinear differential equations, depending on what is chosen as control variables. The instrument chosen to characterize the process control system is for instance the valves, which are often used in their quality of a simple tool for furnishing disturbance to plants. These valves often provide a simple means of modulating the input signals, given in the form of maximum-sequence or other binary signals.

Bilinear models (BM) can approximate a wide class of nonlinear systems. They are used to model nonlinear processes in signal and image processing and communication systems modeling. In particular, they arise in areas such as channel equalization, echo cancellation, nonlinear tracking, multiplicative disturbance tracking, and many other areas of engineering, socioeconomics, and biology. BM represent a mathematically tractable structure over Volterra models for a nonlinear system. Also, a bilinear model can obviously represent

the dynamics of a nonlinear system more accurately than a linear model. Hence, modeling and control of nonlinear systems in a bilinear framework are fundamental problems in engineering.

This chapter proposes new methodologies for analysis and modeling of nuclear power plant reactors as controlled systems using algebraic and geometric methods. These can be subdivided into methods that attempt to treat the system as a bilinear system in a limited range of operation and use bilinear design methods for each region. The most important aspect of these methodologies is transformation of a nonlinear control system into a bilinear system.

The controllability, observability, and invertibility of nonlinear control systems using Lie algebras of vector fields are considered. The study of this type of systems was initiated by R. Brockett [6]. Brockett's observability results are generalized and necessary & sufficient conditions for observability are presented. Effective algorithms are proposed to verify such conditions.

Local and global bilinear realizations of nonlinear control systems were studied in the literature. For a controlled nonlinear system with control appearing linearly, there exist necessary and sufficient conditions for the existence of a dynamically equivalent bilinear system. It was also shown that every nonlinear realization can be approximated by a bilinear realization [16, 17].

This chapter is organized as follows. In Section 2, we discuss the principles of nuclear reactor dynamics. This enables us to formulate a bilinear model describing the nuclear power plant reactor in Section 3. Critical states of the nuclear power plant core are represented via versal models, Section 4. Thermal-hydraulic systems in the reactor core are modeled via bilinear models, Section 5. The coefficients of this bilinear model are obtained via an identification algorithm, discussed in Section 5.1. In Section 6, we discuss the simulation of nuclear power plant reactor core accidents.

2 System-Theoretical Description of Nuclear Reactor Dynamics

Bilinear systems are one of the simplest nonlinear systems and therefore particularly applicable to analysis of much more complicated nonlinear systems. They can be used to represent a wide range of physical, chemical, biological, and social systems, as well as manufacturing processes that cannot be effectively modeled under the assumption of linearity [23, 25, 26].

We emphasize the role of three disciplines that modified our outlook on bilinear system theory. The first one is modern differential geometry. The second discipline is the modern theory of control dynamical systems. The third discipline is optimization theory. Bilinear systems can approximate a wide class of nonlinear control systems. They can be represented as state space models or as systems of input-output).

The wide spectrum of the above-mentioned problems can be represented by the following theoretical schemes.

1. Construction of a set of states accessible from a given initial state
2. Identification of the set of controls steering the system from a given initial state to a desired accessible state with the greatest or specified probability
3. Stability analysis for adaptive bilinear systems
4. Identification of a control that is optimal with respect to a given criterion, for example, the response time or the minimum of switches (in bang-bang control)
5. Control and optimization of nonlinear systems
6. Construction of a system of a feedback providing for the possibility of control with accumulation of data

A global change in coordinates for transforming the system are used for finding a lower-order nonlinear subsystem. A constructive system analysis of such systems on the base of geometric and algebraic methods is conducted. The specific examples of nonlinear systems reduction to bilinear systems (BS) and dynamical systems (DS) with known physical properties are given. It is also shown that every nonlinear realization can be locally approximated by a bilinear realization, with an error that grows as a function of time t.

Necessary and sufficient conditions for the invertibility of a class of nonlinear systems, which includes matrix bilinear systems, were also obtained. Lie algebraic invertibility criteria are obtained for bilinear systems in $\mathbb{R}^n$, which generalize standard tests for single input linear systems. These results are used to construct nonlinear systems that act as left-inverses for bilinear systems.

Due to the widespread use of bilinear models, there is strong motivation to develop identification algorithms for such systems given noisy observations Fnaiech, Ljung and Fliess's paper [9] presents methods for parameter identification of bilinear systems. These methods are directly transferred from linear system identification methods, such as least squares and recursive prediction error methods. A conjugate gradient method for identification of bilinear systems has been developed by Bose and Chen [4]. Most studies of the identification problem of bilinear systems have assumed an input-output formulation. Standard methods such as recursive least squares, extended least squares, recursive auxiliary variable, and recursive prediction error algorithms, have been applied to identifying bilinear systems.

In this chapter we describe new principles of monitoring control, and optimization of a large class of nonlinear objects including nuclear reactor cores [2, 7, 8, 10, 8, 12, 14, 20, 26]. Nonlinear physics and bilinear control are two rich and well-developed theories. Their efficient unification requires joint efforts of specialists in both fields. When two such abundant theories are joined, the effect is multiplicative rather than additive because they amplify each other's potential in proportion to their range of development.

3 Bilinear Logic-Dynamical Models

Suppose that a nonlinear process in the nuclear power plant reactor can be described by equation

$$\dot{y}(t) = b_0(y) + \sum_{i=1}^{h} u_i(t) b_i(y),$$
$$z(t) = f(y(t)), \quad y(0) = y^0, \quad u(t) \in \Omega, \quad y \in Y, \tag{1}$$

where $y = (y_1, \ldots, y_n)$ is a state vector; $z = (z_1, \ldots, z_n)$ is a vector of sensor outputs; $b_0(y), \ldots, b_n(y)$ are analytical vector fields; f is an infinite differentiable $\mathbb{R}^1$ vector-function; Y is a compact manifold, and $u(t) \in \Omega = \{u : |u_i| \leq 1, i = 1, \ldots, h\}$.

By using coordinate transformations we want to construct a logic-dynamical system, *i.e.* a system describing the processes evolving according to continuous dynamics, discrete dynamics, and logic rules.

Consider the system

$$\dot{x}(t) = \sum_{j=1}^{r} L_j \left[A_{0j} + \sum_{i=1}^{h} u_1(t) A_{ij} \right] x(t),$$
$$\omega(t) = \sum_{j=1}^{r} L_j C_j x(t), \quad x(0) = x, \quad u(t) \in \Omega \tag{2}$$

and consider the matrix equation

$$\dot{X}(t) = \left(A_0 + \sum_{i=1}^{h} u_i(t) A_i \right) X(t),$$
$$W(t) = CX(t), \quad X(0) = I, \quad u(t) \in \Omega, \tag{3}$$

where $X(t)$ is a matrix, which evolves in $Gl(m, \mathbb{R})$, of invertible $(m \times m)$ matrices. Each column of this equation is a system in the form (1).

The Lie algebra of the group $Gl(m, \mathbb{R})$ is finite-dimensional over the real field $\mathbb{R}$. There is a closed Lie subgroup G of $Gl(m, \mathbb{R})$ which corresponds to the subalgebra g of the algebra $gl(m, \mathbb{R})$. This algebra is defined by the Lie bracket and the matrices $\{A_0, \ldots, A_h\}$ are characterized by the solution of the equation

$$\dot{X}(t) = \left(\sum_{i=1}^{h} u_i(t) A_i \right) X(t),$$
$$(X(0) = I, \quad |u_i| \leq 1, \quad i = 0, \ldots, h).$$

The group G contains the set of all accessible matrices of (3). The set of accessible matrices of the system is a subset of G with nonempty interior

in the relative topology of G, hence G is the smallest subgroup of $Gl(m,\mathbb{R})$ containing all accessible matrices of (3).

Let S_j be some neighborhood of the point y_j^0; then $W_j(S_j)$ is a minimal subalgebra of the Lie algebra C^∞ of all vector fields on S_j over $\mathbb{R}$ containing $\{b_0,\dots,b_h\}$, and a submanifold Y_j containing y_j^0, is an integral manifold $\widetilde{W}_j(S_j)$, whereas the dimension of Y_j is equal to the rank $\widetilde{W}_j(S_j)$ at the y_j^0. Then, according to Chow's theorem, the set of all points tY_j is accessible by the system (1) from y_j^0.

Because Y is a compact manifold, there exist submanifolds Y_j', such that $Y=\cup_{j=1}^r Y_j'$. If the subalgebra $\widetilde{W}_j(Y_j')$ is finite-dimensional, then there exists a Lie subalgebra g_j of the algebra $gl_j(m_j,\mathbb{R})$ for some m_j, and according to the Ado's theorem (Ado, 1947), an isomorphism of Lie algebras $\varphi_j\colon \widetilde{W}_j(Y_j')\mapsto g_j$. We define the matrix bilinear system (3) by the map $A_{ij}=\varphi_j(b_i)$. Let l_j be the map

$$l_j : W_j(Y_j')\mapsto \widetilde{W}_j(y_j^0),$$

such that $l_j(c)=c(y_j^0)$ for $c\in\widetilde{W}_j(Y_j')$. Then the linear map $l_j'=l_j\circ\varphi_j^{-1}$ satisfies the condition

$$l_j'=\big(\big[A_{i_1j}\dots\big[A_{i_{\nu-1}j},A_{i_\nu j}\big]\dots\big]\big)=\big[b_{i_1j}\dots\big[b_{i_{\nu-1}j},b_{i_\nu j}\big]\dots\big](y_j^0)$$

for any ν_i, $0\le i_1,\dots,i_\nu\le h$. By Krener's theorem [17], there exists a neighborhood M of I and maps $\lambda_j\colon M_j\mapsto Y_j'$, that preserve the solutions.

By Brockett's theorem [6], we can find the following result. If the equation (1) satisfies the above stated conditions and the map $f\circ\lambda_j\colon X\mapsto Z$ is polynomial, then there exists a logic-dynamical realization (2) of $u(t)\mapsto\omega(t)$ and a constant $T\ge 0$, such that for any input $u(t)$, the corresponding outputs satisfy $\omega(t)=z(t)$ for $t\in[0,T]$.

Remark 0.1 The dimension of a state space of LDS is the maximal dimension of Euclidean space, corresponding to some submanifold M_j.

We define a logic variable L_j for each integral submanifold Y_j' of the compact space state Y by the following;

$$L_j=\begin{cases}0, \text{ if } y\in Y_j', & j=1,\dots,r,\\ 1, \text{ if } y\notin Y_j' & \text{otherwise.}\end{cases}\tag{4}$$

We suppose that the logic function L_j can be realized by a finite automaton. For each value $z_i\in Z$, $i=1,\dots,r$ we can find a submanifold Y_t by the map $\gamma_t\colon T\times Y\mapsto Z$. This map satisfies the condition

$$\gamma_t(Y_j')=z_j,\quad Y_i'\cap Y_j'=\phi,\quad i\ne j.$$

If the system (1) satisfies the above hypothesis, then there exists a logic-dynamical system (2), such that for any input $u(t)$, the corresponding outputs satisfy $z(t)=\omega(t)$, $t\in[0,T]$.

4 Versal Models of Critical States

Mathematical model of critical states in nuclear plant core can be described by versal or universal models. The concept of a versal or universal mapping was introduced in Arnold [1], however, the methods for calculation of the parameters of a versal or universal model using an initially given model of a time-varying system are important for engineering applications. In other words, the case in point is the construction of analytical dependence of parameters of a universal model as a function of parameters of an a priori given model, *e.g.* of its controlling part. This problem can also be interpreted as the problem of robust decomposition of sets of dynamical systems. It should be pointed out that each subsystem forming a part of the universal model contains a minimum admissible number of parameters from the point of view of completeness of consideration of possible variants of subsystem interaction in the initial model and admits an independent investigation. In this case, interaction between the subsystems in the initial model is reduced to parametric interaction (self-operation) in these subsystems. Interactions between the initial subsystems that cannot be removed in this way appear only in the cases where there are singularities in the initial subsystems (symmetry, close eigenfrequencies, singularity of the matrix of higher derivatives of differential equations in the initial model, and possibly some others). In addition to the circumstances mentioned above, selection of dimension of universal subsystems is determined by computing resources used for calculation of parameters of universal models from preset interaction coefficients and for investigation of the models themselves. Once such dependencies are obtained, investigation of a universal model becomes practically manageable and can be easily performed analytically.

Let us point out also that the construction of a universal model admits its extension by connecting new subsystems. In this case, algorithms for calculation of universal model parameters are arranged so that they allow us to refine the parameters of the initial universal model with regard to the presence of new subsystems and, at the same time, to determine parameters of the universal model of the connected subsystem as a function of the initial varied parameters of the whole system.

Methods for calculation of versal model parameters based on the Campbell–Hausdorff decomposition are well known.
Let $A = A_0 + B$, where A_0 is the constant principal matrix of the object, B is the matrix of the interaction constant or is analytically depending on the parameters. We apply to the matrix A, the homothetic transformation e^S parameterized by means of a matrix exponential curve and obtain

$$\widehat{A} = e^{-S} A e^S = e^{-S}(A_0 + B)e^S = A_0 + X.$$

The matrices S and X should be determined from the known matrix B.

Let us consider a formal expansion of S and X in terms of degrees of the matrix B:

$$S = S^1 + S^2 + \cdots, \quad X = X^1 + X^2 = \cdots,$$

where the superscript is the exponent of the expansion with respect to B. To obtain the component of this expansion, we expand the matrix A into the Campbell–Hausdorff series:

$$\widehat{A} = A_0 + X = e^s A e^s = A + [A, S] + \frac{1}{2!}[[AS]S] + \frac{1}{3!}[[[AS]S]S] + \cdots,$$

where $[A, S] = AS - SA$ is a Lie bracket. We substitute the expansions of the matrices S and X into this expansion and obtain an infinite system of relations by comparing the terms with equal indices of homogeneity:

$$[A_0 S^1 + B^1] = X^1, \quad B^1 \equiv B,$$
$$[A_0 S^2] + [B^1 S^1] + \frac{1}{2}[[A_0 S^{-1}]S^1] = X^2,$$
$$[A_0 S^3] + [B^1 S^2] + \frac{1}{2}[[A_0 S^2]S^1] + \frac{1}{2}[[B^1 S^1]S^1] + \frac{1}{2}[[A_0 S^1]S^2] = X^3.$$

The formal algorithm for the solution of these equations with respect to the homogeneous components S^i and X^i can be described as follows:

1. Select the first-degree component S^1 in such a way that a maximum number of terms of nonzero elements of the matrix B^1 are annihilated and then determine the first-degree component X^1; the known component $[B^1 S^1] + \frac{1}{2}[[A_0 S^1]S^1]$ appears in this case in the second-degree equations,
2. Select the component S^2 of the transformation so as to annihilate a maximum number of elements in the appeared component and then determine the second-degree component X^2.

The same method should be applied to the third-degree components by selecting S^3, and so on. The algorithm of the transformation e^s is reduced to compensation of as many as possible degrees of perturbation of B, and thus, to decrease its influence in the transformed matrix A. As a whole, this process turns out to be infinite. If we terminate it in N steps, then the terms of degree $N + 1$ and higher with respect to B will remain in the transformed matrix, which symbolically can be written as

$$e^{-s}(A + B)e^s = A_0 + X \pmod{B^{N+1}}.$$

A practical implementation of this algorithm is difficult, inasmuch as it is not clear how to perform its first step.

Based on the versal model theory, an alternate, more constructive algorithm can be proposed for calculation of the transformation e^s and the component X that is not annihilated in principle by this transformation.

Essentially it can be reduced to the solution of equations obtained from the Campbell–Hausdorff expansion, simultaneously for the matrices S and X, using the structure of these matrices known from the versal model theory. In

other words, we search for the matrices S in the form of expansion in terms of the base $\{S\}$ from matrices transversal to the centralizer of the matrix A_0:

$$S = \sum_{i=1}^{m} \omega_i S_i \equiv S^1 + S^2 + \cdots + S^m.$$

The basic matrices S, for different types of the matrices A_0 can be constructed in an explicit form. We search for the matrices X in the form of $\{x_k\}$-base expansion of the normal to the orbit:

$$X = \sum_{k=1}^{p} \lambda_k X_k \equiv X^1 + X^2 + \cdots + X^p, \quad p = n^2 - m.$$

Let us point out that each matrix of the infinite sequences of the matrices $S^1, S^2, \ldots$ (or $X^1, X^2, \ldots$) can be decomposed in terms of a finite base $\{S_i\}$ or $\{X_i\}$, respectively.

If the matrix B is given numerically, then we have the following system of equations for determination of the homogeneous components S^i and X^i from the Campbell–Hausdorff expansion,

$$\begin{gathered} X^1 - [A_0 S^1] = B^1 \equiv B, \\ X^2 - [A_0 S^2] = B^2 = [B^1 S^1] + \frac{1}{2}[[A_0 S^{-1}] S^1], \\ X^3 - [A_0 S^3] = B^3 = [B^1 S^2] + \frac{1}{2}[[A_0 S^2] S^1] \\ + \frac{1}{2}[[B^1 S^1] S^1] + \frac{1}{2}[[A_0 S^1] S^2], \end{gathered}$$

which can be solved recurrently. With a given structure of the matrices S^i and X^i, each equation of this system is of the same type and they differ only by their right-hand sides. A solution of each equation can be obtained by parts using a block representation of the matrices A, S^i, and X^i. The required result is obtained through summation of a finite number of the matrices S^i and X^i with the selected degree N of homogeneity.

Let us consider the algorithm of construction of the solution in the form of an explicit dependence on varied parameters. Let the matrix B of dimension $(n \times n)$ be a linear function of parameters

$$B(\mu) = \sum_{i=1}^{S} \mu_i B_i, \quad S \le n^2,$$

where B_i are constant matrices.

We present homogeneous components of the matrices X and S in the form

$$X^1 = \sum_{j=1}^{s} \mu_j Y_j, \quad X^2 = \sum_{j,k=1}^{s} \mu_j \mu_k Y_{jk},$$
$$X^3 = \sum_{j,k,l=1}^{s} \mu_j \mu_k \mu_i Y_{j,k,i}, \ldots,$$
$$S^1 = \sum_{j=1}^{s} \mu_j Q_j, \quad S^2 = \sum_{j,k=1}^{s} \mu_j \mu_k Q_{jk},$$
$$S^3 = \sum_{j,k,l=1}^{s} \mu_j \mu_k \mu_i Q_{j,k,i}, \ldots, \tag{5}$$

where $Y_j, Y_{jk}, \ldots, Q_j, Q_{jk}, \ldots$ are two infinite sequences of matrices from finite-dimensional spaces $X = \{X_1 \ldots X_p\}$ and $S = \{S_1 \ldots S_m\}$.

Having substituted these expansions into the Campbell–Hausdorff expansion, we obtain the equations for determining the matrices Y_j, Q_j, Y_{jk}, and Q_{jk}:

$$Y_j - [A_0 Q_j] = B_j,$$
$$Y_{jk} - [A_0 Q_{jk}] = B_{jk} = [B_j Q_k] + \frac{1}{2}[[A_0 Q_j] Q_k],$$
$$Y_{jki} - [A_0 Q_{jkl}] = B_{jkl} = [B_j Q_{kl}] + \frac{1}{2}[[A_0 Q_{kl}] Q_j]$$
$$+ \frac{1}{2}[[B_j Q_k] Q_l] + \frac{1}{2}[[A_0 Q_l] Q_{kl}],$$
$$j, k, l = 1, \ldots, S.$$

Because the spaces of the matrices X and S are of finite dimension, each of the two infinite sequences of the matrices $\{Y_j, Y_{jk}, \ldots\}$ and $\{Q_j, Q_{jk}, \ldots\}$ is a finite-dimensional linear combination of the basic sequences:

$$Y_j = \sum_{q=1}^{p} a_{jq} X_q, \quad Y_{jk} = \sum_{q=1}^{p} a_{jkq} X_q, \quad Y_{jkl} = \sum_{q=1}^{p} a_{jklq} X_q,$$
$$Q_j = \sum_{r=1}^{m} b_{jr} S_r, \quad Q_{jk} = \sum_{r=1}^{m} b_{jkr} S_r, \quad Q_{jklr} = \sum_{r=1}^{m} b_{jklr} S_r, \tag{6}$$

where $\{a_{iq}, a_{jkq}, \ldots\}$, $\{b_{ir}, b_{jkr}, \ldots\}$ are constant coefficients that can be calculated from the systems of linear algebraic equations of the type

$$\sum_{q=1}^{p} a_q Sp X_q X_{q'}^* = Y, \quad q' = 1, \ldots, p,$$
$$\sum_{r=1}^{m} b_r Sp S_r S_{r'}^* = Q, \quad r' = 1, \ldots, m,$$

after substitution of the matrices $\{Y_j, Y_{jk}, \ldots\}$ for the coefficients $\{a_j, a_{jq}, \ldots\}$ and matrices $\{Q_j, Q_{jk}, \ldots\}$ for the coefficients $\{b_j, b_{jk}, \ldots\}$ into their right-hand sides.

Having substituted expansions (6) into expansion (5), we obtain expressions for the parameters of the universal model in the form of power series in parameters of the initial strain:

$$\omega_r(\mu) = \sum_{j=1}^{s} b_{jr}\mu_j + \sum_{j,k=1}^{s} b_{jkr}\mu_j\mu_k + \sum_{j,k,l=1}^{s} b_{jklr}\mu_j\mu_k\mu_l + \cdots,$$

$$\lambda_q(\mu) = \sum_{j=1}^{s} a_{jq}\mu_j + \sum_{j,k=1}^{s} a_{jkq}\mu_j\mu_k + \sum_{j,k,l=1}^{s} f_{jklq}\mu_j\mu_k\mu_l + \cdots.$$

If we restrict ourselves to the terms of the Nth degree in these series, then we can speak about universal models of the orders $1, 2, \ldots, M$.

5 Bilinear model of the thermal-hydraulic systems

The dynamics of a reactor core channel of nuclear plants may be represented by a set of ordinary differential equations. For system analysis of critical states associated with the channel, three cases can be considered in respect of the variable to be modulated:

(a) The inlet coolant temperature,
(b) the reactivity and
(c) the coolant flow rate.

In case (a), the dynamics become linear if we choose the inlet coolant temperature as identification input with the other variables considered fixed. The same applies to case (b), but the neutron dynamics require to be known when the control rods are displaced to change the reactivity. In case (c), the dynamics become bilinear, if besides the above the coolant flow rate is chosen as a reference input to identify the parameters. Now, for practical considerations, the reference input designated for case (a) is not very convenient in actual implementation. Case (b) has been widely utilized in the past for reactor identificationand particularly for examining reactor core dynamics. In the last-mentioned case, however, it is not desirable to apply reactivity changes of large amplitude, which are liable to impair the neutron flux balance.

A mathematical model of a channel in a reactor core can be represented by the the ordinary differential equations

$$\begin{aligned} \dot{x}_1 &= -2h(x_1 - x_2)/\rho ca + p/\rho c, \\ \dot{x}_2 &= -2ah(x_1 - x_2)/(b_2 - a_2)de - v(x_{2|_{y=L}} - x_{2|_{y=0}}/L), \end{aligned} \tag{7}$$

where x_1 is the average temperature of fuel pin; x_2 is the average temperature of coolant; p is the average power; ρ is the density of fuel pin; a is the radius

of el pin; d is the density of coolant; e is the specific heat of coolant; a is the radius of fuel pin; L is the core height; c is the specific heat of fuel pin; h is the heat transfer coefficient; b is the radius of coolant flow channel tube; v is the coolant flow velocity.

In deriving the above equation the following assumption have been used:

(a) no boiling;
(b) the heat conduction through coolant flow along the fuel pin neglected.

Now, letting

$$x_2 = (x_{2|_{y=L}} + x_{2|_{y=0}})/2, \tag{8}$$

and with the outlet coolant temperature $x_{2|_{y=L}}$ as output and the coolant flow velocity v as input, we obtain the bilinear equation

$$\begin{aligned}
\dot{x}_1 &= -2h(x_1 - x_2)/\rho ca + p/\rho c,\\
\dot{y}_2 &= -\lambda_2 y_2 + y_1,\\
\dot{y}_3 &= -\lambda_2 y_3 + v y_1,\\
\dot{y}_4 &= -\lambda_2 y_4 + u_2,\\
\dot{y}_5 &= -\lambda_2 y_5 + u_1 v,
\end{aligned} \tag{9}$$

$$\begin{aligned}
z_1 &= y_1 - \lambda_2 y_2,\\
z_2 &= v(y_2 - u_1),\\
z_3 &= y_3 - y_5,\\
z_4 &= -z_4 - y_4,\\
z_5 &= 2y_1 - u_1.
\end{aligned} \tag{10}$$

We can show that the bilinear equation (5) is equivalent to (7) with initial condition

$$y_1(0) = y_{10}, \quad y_2(0) = y_{20}, \quad y_3(0) = y_4(0) = y_4(0) = 0. \tag{11}$$

Next section describes an algorithm which provides stable parameter identification and adaptive control for the thermo-hydraulic system described by the bilinear differential equation (5).

5.1 Identification algorithm

In this section we describe an identification method based on the expansion of signal processes over an orthogonal basis. Using this methodology we can obtain a system of linear algebraic equations, which is used to determine the coefficients of the bilinear model. By means of the least squares method we obtain estimates of the unknown parameters of the model. It is based on a discrete approximation of the input-output map of a nonlinear object [24].

Consider the bilinear model

$$\dot{x}(t) = Ax(t) + Lu(t) + \sum_{j=1}^{n} B_j x(t) u_j(t), \tag{12}$$

where A, L, and B_j are unknown parameters to be estimated; u is a control. By the generalized product of orthogonal series we mean

$$u_j(t) = \sum_{t=0}^{m-1} u_{jl} t^l,$$

$$x(t)u_j(t) = \sum_{l=0}^{m-1} u_{jl} X t^l \Pi(t) = \sum_{l=0}^{m-1} u_{jl} X R_l \Pi(t).$$

The integration of (8) gives

$$x(t) - x(0) = A \int_0^t x(t')dt' + L \int_0^t u(t')dt' + \sum_{j=1}^{n} L_j \int_0^t x(t')u_j(t')u_j(t')dt'. \tag{13}$$

Using this result, we obtain

$$X\Pi - X(0)\Pi = AXE\Pi LUE\Pi + \sum_{j=1}^{n} B_j X \left[\sum_{t=0}^{m-1} u_{jl} R_l\right] E\Pi. \tag{14}$$

Substituting the expression for Θ into (14) gives

$$XG\Pi - \sum_{j=1}^{n} X(0)G\Pi = AXEG\Pi + LUEG\Pi + \sum_{j=1}^{n} B_j X[\sum_{j=1}^{m-1} u_j R_j]EG\Pi(t)$$

or

$$XG - X(0)G = AXEG + LUEG + \sum_{j=1}^{n} B_j X[\sum_{l=0}^{m-2} u_{jl} R_l]EG,$$

$$ZS = (X - X(0))G, \tag{15}$$

where Z is the parameter vector; that is,

$$Z = [ALB_1 B_2 \dots B_n]. \tag{16}$$

6 Bilinear Simulation of Reactor Core Accidents

The power increase in the reactor core during a control system accident can be effectively described by bilinear model in terms of reactivity released, the Doppler reactivity feedback coefficient, the delayed neutron fraction and the lifetime of prompt neutrons [8, 21]. Given the speed of the transient, two assumptions have to be made:

1. Energy cannot be transferred from fuel to water, and
2. there is no time for delayed neutrons to be emitted.

These assumptions match the adiabatic point model and apply when reactivity is very high. When feasible, these assumptions provide good illustrations to safety problems demonstration. The neutron bilinear balance equation (17) is

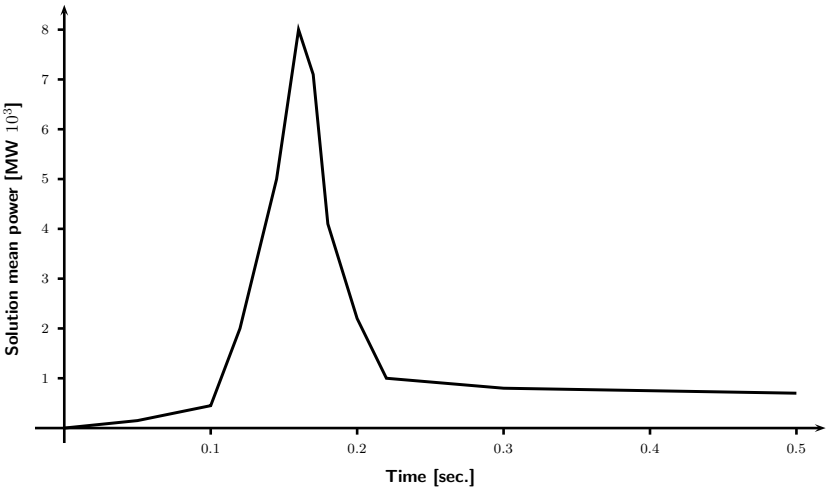

Fig. 1. Core power increase. The dependence of the average power of the reactor containment on time

expressed simply as

$$\dot{x} = \frac{\rho(t) - \beta}{\lambda} x(t), \tag{17}$$

with λ representing prompt lifetime, and β being the delayed neutrons fraction [19, 3].

The increase in reactivity

$$\rho(t) = \rho_0 - \alpha \left(T(t) - T(0)\right) \tag{18}$$

is expressed in terms of the total reactivity ρ_0 of the control rods assembly, while the Doppler reactivity feedback effect of the fuel is expressed through the coefficient α and the mean temperature increase in the core T.

The mean temperature increase in the core T depends directly on the energy produced

$$M \cdot C\,(T(t) - T(0)) = \int_0^1 x(\tau)d\tau, \tag{19}$$

where M represents the mass and C is the specific heat of the fuel.

It is also demonstrated that the produced energy E and the maximum power $x_{\max}$ reached during the transition satisfy the expressions [2]

$$E = 2M \cdot C\frac{\rho_0 - \beta}{\alpha} \tag{20}$$

and

$$x_{\max} = M \cdot C\frac{(\rho_0 - \beta)^2}{\alpha\gamma}. \tag{21}$$

The results depend on the estimation of the constants used in the mathematical applications. It can also be directly demonstrated that the reactivity at maximum power equals β. In this way, the key aspects of the power increase, see Figure 1, during the first moment of the transition can be obtained. For example, important variables for studying the thermo-mechanical behavior of the fuel, such as the rate of power increase and the width of the curve $x(t)$ are obtained explicitly.

7 Conclusions

In this chapter, we consider the problem of determining critical states of nuclear plant reactors using bilinear modeling. Mathematical bilinear modeling and numerical analysis of initial critical events are proposed. We considered the ways in which the disposition of the phase curves of a vector field of the dynamical model can alter in a neighborhood of a singularity as the parameters on which the vector field depends vary. A technical convenience in the study of such changes are certain deformations having a special universality property - the so-called versal families. Our results are presented mainly in the form of explicit formulae for versal families and an analysis of the corresponding bifurcation diagrams.

A bilinear model of thermo-hydraulic system is proposed. We described an identification method based on the expansion of signal processes over an orthogonal basis. Using this methodology we can obtain a system of linear algebraic equations, which is used to determine the coefficients of the bilinear

model. By means of the least squares method we obtain estimates of the unknown parameters of the model. The computational algorithm obtained has quite good accuracy. An algorithm for identification of the bilinear discrete models is obtained. It is based on a discrete approximation of the input-output map of a nonlinear object.

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