Short Term Portfolio Optimization for Discrete Power Plant Dispatching

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***Abstract*—We consider a public power service in the liberalized market, operating its own power plant and participating in the spot market. In the short-term horizon, the objective is to optimize the dispatch of the power plant in discreet steps. The public service has to meet the demand of its customers, while trading in the spot market where transaction costs apply. The power plant operation is subject to several technical constraints, as minimum up- / down-time, ramping constraints, minimum operation level constraints, minimum time operational level constraints, minimum / maximum production per day while the costs are given by the start-up costs, the operational cost, following an efficiency curve, and CO2 emission cost. This problem is modeled through a mixed integer linear programming formulation.**

***Index Terms*—balancing market, CO2 certificates, mixed integer linear programming, portfolio optimization, spot market, power plant dispatch, revenue maximization, transaction costs**

I. Nomenclature

The nomenclature of this article is summarized in Table I and Table II. All variables in the model have small letters while the input data and coefficients are given in capital letters.

II. Introduction

We discuss a day-ahead portfolio optimization problem of an electric utility in the liberalized market. The utility has to meet the demand of its customers. Therefore, the electricity provider can use its own power plant, buy / sell power at the spot market and balancing market, as well as purchase power from a load following contract. The trading in the spot market is considered in hourly time steps, while the balancing market and the load following contracts are quarter-hourly.

It is important to mention that the power plant can only be operated in discrete steps. This is necessary, as an operator would never choose an infinite continuum of steps but rather a small number of usual operating points, which are called *partial load operation points*. They are determined by the technical attributes of the power plant and are supposed to be given; determining the number of discrete steps the plant can be operated. These discrete operational levels are called *states* of the power plant and the state where the power plant is shut down is called *idle state*.

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TABLE I
Variables Used In The Model

Symbol	Variable Type	Unit	Meaning
δ_{mt}	binary	–	Indicate if power plant is in state m at time t
χ_t^S	non-neg. cont.	–	Indicate if the power plant is shut down at the beginning of period t
χ_t^I	non-neg. cont.	–	Indicate if the power plant left the idle state at the beginning of period t
χ_t^C	non-neg. cont.	–	Indicate a state change at the beginning of period t
p_t^{PP}	non-neg. cont.	MW	Power produced by the power plant in period t
$p_h^{\mathrm{PM,b}}$	non-neg. integer	MW	Power bought at the power market in period t
$p_h^{\mathrm{PM,s}}$	non-neg. integer	MW	Power sold at the power market in period t
$p_t^{\mathrm{BM,b}}$	non-neg. cont.	MW	Power bought at the balancing market in period t
$p_t^{\mathrm{BM,s}}$	non-neg. cont.	MW	Power sold at the balancing market in period t
p_t^{LF}	non-neg. cont.	MW	Power bought from the load following contract in period t

Furthermore, the power plant operation underlies a variety of technical restrictions. When the operation of the power plant is started, then it has to run for a minimum time horizon in any of the operational levels, except the idle state. This constraint is called *minimum up-time*. Once the power plant is shut down, it has to remain in the idle state for a certain time period, called *minimum down-time*. The switching of the power plant levels is represented by the *ramping constraints*. Furthermore, the power plant cannot be operated below a certain threshold level, except if it is shut down. We call this restriction *minimum operational level constraint*. Our model allows us to restrict the power plant to change the operational level within certain time horizon, we call this constraint *minimum time operational level constraint*. In addition, the electricity generation of the own power plant for the whole day is restricted by a lower and an upper bound. Finally, the *unit commitment constraints* give the initial condition of the power plant at the beginning of the day-ahead planning.

Transaction costs have to be considered in the appropriate way for each of the buy / sell options as they have a direct impact on the operation schedule. The latter occurs from the choice between allowing separated access for buying and selling from the market for the generation and the client portfolio or aggregating the position of the two portfolios first

TABLE II
DATA / COEFFICIENTS USED IN THE MODEL

Symbol	Unit	Meaning
P_t	MW	Power demand (forecast) for each quarter-hour time period t
L_m	MW	Power production level of power plant corresponding to state m
F_m^{PP}	–	Efficiency coefficient of the power plant corresponding to state m; not the idle state
C^{PP}	€ /MWh	Variable production cost of the power plant under optimal power plant operation
$C_{\mathrm{SU}}^{\mathrm{PP}}$	€	Start-up cost of the power plant
$D_{\mathrm{UP}}^{\mathrm{PP}}$	–	Minimum up-time of the power plant
$D_{\mathrm{DO}}^{\mathrm{PP}}$	–	Minimum down-time of the power plant
$D_{\mathrm{OP}}^{\mathrm{PP}}$	–	Minimum time of the power plant to operate at the same level
$E_{\min}^{\mathrm{PP}}$	MWh	Minimum electricity production of the power plant per day
$E_{\max}^{\mathrm{PP}}$	MWh	Maximum electricity production of the power plant per day
P_h^{PM}	€/MWh	Hourly electricity price in the power market
$P_{\mathrm{T}}^{\mathrm{PM}}$	€/MWh	Transaction costs for the trading in the power market
$P_t^{\mathrm{BM,b}}$	€/MWh	Price for the power bought from the balancing market during time period t
$P_t^{\mathrm{BM,s}}$	€/MWh	Price for the power sold at the balancing market during time period t
P^{CO2}	€/t CO2	CO2 emission price
F_m^{CO2}	t CO2/MWh	Emission coefficient of the power plant dependent on state m
F_h	h	Coefficient converting power to electricity for one hour interval
F_t	h	Coefficient converting power to electricity for one quarter-hour interval

and then allowing access to the market. The first option is applicable when considering transaction costs where in the second option transaction costs are neglected.

In this model, we assume all data to be given explicitly. More specifically, we assume that we know the electricity price in the spot market. Bidding models such as the one presented in [1] take this uncertainty into account. Furthermore, the purchase and sales price in the balancing market is assumed to be deterministic.

The model formulation presented in this article is based on [2]; especially the modeling of the discrete power plant dispatching. The electricity optimization problem presented in this article belongs to the class of *unit commitment* and *economic dispatch problems*. In contrast to the unit commitment problems, this model does not include any constraints on the power transmission.

In [3] the authors provide a mixed integer linear programming (MILP) formulation of the unit commitment problem, while taking into account energy exchange contracts. The model presented in [4] for thermal plants uses less binary variables than the model presented in [3]. The model presented in this paper assumes a discrete cost structure for the power plant in contrast to the quadratic one discussed by [4]. Mixed integer programming was also used in [5] to solve the unit commitment problem. The optimal selling of energy in the electricity spot market is modeled as an MILP problem in [6] and as a stochastic program in [7]. In the literature, there are many specialized algorithms for solving the unit commitment problem [8]–[12] as well as the economic dispatch problem [3], [13], [14].

In Section III, we present a mixed integer linear programming formulation for the day-ahead portfolio optimization problem. The constraints and model properties are discussed in detail. Generalizations and modifications are considered in Section IV. We conclude with Section V.

III. MODEL FORMULATION

Located in the liberalized market, the electric utility wants to maximize its profits while meeting the demand of its customers.

Following common standards, the day-ahead planning has a resolution of quarters of an hour [15], [16]. This leads to 96 quarter-hour time intervals per day

$$t \in T := \{1, \ldots, N^{\mathrm{T}} = 96\} \quad . \tag{1}$$

The power demand of the customers is given as a forecast of electric power for the next day for each of the time periods t as

$$P_t \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} \quad , \tag{2}$$

measured in MW.

The utility's power portfolio consists of its own power plant, the spot market, balancing market and a load following contract.

A. Power Generation

For practical reasons, the power plant can only be operated in discrete steps. Those states of the power plant are indexed by

$$m \in M := \{1, \ldots, N^{\mathrm{M}}\} \quad , \tag{3}$$

where N^{M} is the number of operational levels. We associate state $m = 1$ with the idle state. Typically, a power plant has around $N^{\mathrm{M}} = 8$ states and can be operated in 10% steps [2]. The states can be equidistant or not, dependent on the power plant and the modeling purpose.

The discrete operation levels of the plant are modeled via the binary variables δ_{mt}, having value 1, if the power plant is in state m at time period t and 0 otherwise.

Let L_m be the power production level of the power plant operated at state m. Then, the generated power in MW of the power plant for each time t is given by

$$p_t^{\mathrm{PP}} = \sum_{m=1}^{N^{\mathrm{M}}} L_m \delta_{mt} \quad . \tag{4}$$

B. Trading in the Spot Market

The utility can buy / sell power in the spot market every hour at a discrete quantity. In Europe, the European Energy Exchange (EEX) in Leipzig provides such a spot market [17]. The power products of interest here are *hourly contracts* traded at one day and delivered at the next day. The price P_h^{PM} in € per MWh is assumed to be known. A fixed transaction costs of $P_{\mathrm{T}}^{\mathrm{PM}}$ in € per MWh applies for each transaction.

Let $p_h^{\mathrm{PM,s}}$ be the discrete power in MW sold in the spot market and $p_h^{\mathrm{PM,b}}$ be the discrete power in MW bought at each hour h within the 24 hours planning horizon. Then, the total revenue from the electricity traded in the spot market per day is given by

$$\sum_{h=1}^{24}\left(P_h^{\mathrm{PM}}-P_{\mathrm{T}}^{\mathrm{PM}}\right)F_h p_h^{\mathrm{PM,s}}-\sum_{h=1}^{24}\left(P_h^{\mathrm{PM}}+P_{\mathrm{T}}^{\mathrm{PM}}\right)F_h p_h^{\mathrm{PM,b}} \quad , \tag{5}$$

where $F_h = 1$ [h] measured in hours is the coefficient converting power [MW] for one hour to electricity [MWh].

Recognize that the transaction costs are a fixed fee, determining the difference between the buy and sell price in the spot market.

C. Balancing Market

The balancing market is an important mechanism in the electricity market as its role is to cover all load deviations in real-time [18]. The balancing market and the load following contract are the two flexible instruments that offer the possibility to follow the contract at quarter-hour intervals.

Let $p_t^{\mathrm{BM,s}}(p_t^{\mathrm{BM,b}})$ be the power in MW sold (bought) in the balancing market for a price of $P_t^{\mathrm{BM,s}}(P_t^{\mathrm{BM,b}})$ in € per MWh for each time period t. Recognize that the variables $p_t^{\mathrm{BM,s}}$ and $p_t^{\mathrm{BM,b}}$ are non-negative, continuous. Then, the revenues associated with the trading in the balancing market are

$$\sum_{t=1}^{96} P_t^{\mathrm{BM,s}} F_t p_t^{\mathrm{BM,s}} - \sum_{t=1}^{96} P_t^{\mathrm{BM,b}} F_t p_t^{\mathrm{BM,b}} \quad , \tag{6}$$

where $F_t = 0.25$ [h] measured in hours is the coefficient converting power [MW] for a quarter-hour time interval to electricity [MWh].

D. Load Following Contract

The load following contract is a contract with a supplier, allowing the utility to purchase power at any time at any given quantity offering thus flexibility. This flexibility has to be paid, making this energy source relatively expensive. The load following contracts are also called *full requirements contracts*.

The load following contracts are two-component supply-contracts [19]. The power level peak as well as the delivered energy amount has to be paid. Hence, the cost for the load following contract is the sum of the *power rate*, given in € per MW, and the *energy rate*, given in € per MWh.

The power rate of the load following contract is based on the highest subtraction of power in any quarter-hour time interval within a whole year; one usually applies the arithmetic mean of the two / three highest monthly peaks to avoid random anomalies.

The energy rate is scaled in different annual quantity zones, where a cheaper price per MWh has to be paid if more energy is purchased throughout the year.

The annual price based system makes it difficult to incorporate this instrument into a daily model. The basic idea is to mimic the annual zones in a daily basis. We do not go into any more detail here but instead we refer to [2] for the modeling. For the purpose of this article it is enough to define p_t^{LF} as the power in MW purchased from the load following contract at a price in € determined by function $P_t^{\mathrm{LF}}(p_t^{\mathrm{LF}})$, which can be modeled in the mixed integer programming framework [2].

E. Demand Constraints

The electric utility has to meet the electric power demand for each quarter-hour with its available power portfolio. That gives us

$$p_t^{\mathrm{PP}}+p_t^{\mathrm{LF}}+p_h^{\mathrm{PM,b}}+p_t^{\mathrm{BM,b}}=P_t+p_h^{\mathrm{PM,s}}+p_t^{\mathrm{BM,s}} \quad , \qquad t=1,\ldots,N^{\mathrm{T}} \quad , \quad h=\left\lceil \frac{t}{4}\right\rceil \quad . \tag{7}$$

The flexibility of the load following contracts together with the balancing market allows the utility to meet the demand for all quarter-hour time intervals exactly.

Recognize that due to the positive transaction costs (and because in this model there are no minimum and maximum restrictions on the purchases of power from the spot market), at most one of the variables $p_h^{\mathrm{PM,s}}$ and $p_h^{\mathrm{PM,b}}$ is positive for any given hour. That makes constraint

$$p_h^{\mathrm{PM,s}} p_h^{\mathrm{PM,b}} = 0 \quad , \qquad h=1,\ldots,24 \tag{8}$$

redundant. The same argument holds for the balancing market, as the buy price and the sell price are different.

F. Power Plant Operation Constraints

The power plant has to satisfy a series of constraints.

1) Minimum Up-Time: In order to avoid a shut down of the power plant after a short time period in operation, the power plant has to remain in one of the states, different from the idle state, for at least $D_{\mathrm{UP}}^{\mathrm{PP}}$ quarter-hours. A typical value is 2 hours or $D_{\mathrm{UP}}^{\mathrm{PP}} = 8$. This constraint is called *minimum up-time constraint* and it is related to the plant's technical characteristics: The *start up-* and *shut down-losses* as described in [20] are not considered in this article.

According to [2], let the binary variables χ_t^{S} keep track, if the power plant is shut down at the beginning of time period t

$$\chi_t^{\mathrm{S}} \geq \delta_{1t}-\delta_{mt-1} \quad , \qquad \forall m, \quad t=2,\ldots,N^{\mathrm{T}} \quad . \tag{9}$$

This allows us to formulate the condition

$$\sum_{k=1}^{D_{\mathrm{UP}}^{\mathrm{PP}}} \chi_{t+k-1}^{\mathrm{S}} \leq 1 \quad , \qquad t=1,\ldots,N^{\mathrm{T}}-\left(D_{\mathrm{UP}}^{\mathrm{PP}}-1\right) \quad , \tag{10}$$

ensuring that the power plant can be shut down after at least $D_{\mathrm{UP}}^{\mathrm{PP}}$ time intervals.

As reported in [2], the binary variables χ_t^{S} can be relaxed to be non-negative continuous due to the binary variables δ_{mt} together with (9) and (10). This leads to significant computational improvements.

2) Minimum Down-Time: In analog to the minimum up-time requirement, the power plant has to stay at least $D^{\mathrm{PP}}_{\mathrm{DO}}$ quarter-hours in the idle state, once the power plant is shut down. A typical value can be 4 hours or $D^{\mathrm{PP}}_{\mathrm{DT}} = 16$ time intervals.

Introducing the binary variables χ^{I}_t, indicating if the power plant left the idle state at the beginning of period t

$$\chi^{\mathrm{I}}_t \geq \delta_{1t-1} - \delta_{1t} \quad , \qquad t = 2, \ldots, N^{\mathrm{T}} \quad . \tag{11}$$

The minimum down time can then be modeled as

$$\sum_{k=1}^{D^{\mathrm{PP}}_{\mathrm{DO}}} \chi^{\mathrm{I}}_{t+k-1} \leq 1 \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} - \left(D^{\mathrm{PP}}_{\mathrm{DO}} - 1\right) \quad . \tag{12}$$

Similar to the variables χ^{S}_t, the binary variables χ^{I}_t can be relaxed to be non-negative continuous [2].

3) Ramping Constraints: The power level of the power plant does not change promptly. Hence, we do not allow the power plant to change from any level to an arbitrary one. This is ensured by the hard constraint

$$\delta_{mt-1} + \delta_{nt} \leq 1 \quad , \qquad t = 1, \ldots N^{\mathrm{T}} \quad , \tag{13}$$

for all power plant states changes from state m to n which are forbidden in a single time period; or alternatively, within a quarter-hour interval.

4) Minimum Operational Level Constraint: The power plant should not be operated with less than a certain percentage of its maximal capacity. This is not due to a technical restriction but might be given by the power plant operator; *i.e.*, because the maintenance cost increase. This requirement can be modeled straight forward with the states *m*. For instance, the second state might represent the minimum desired operation level of the plant.

5) Minimum Time Operational Level Constraint: In order to avoid permanent changes of the power plant level, any power state has to continue for at least $D^{\mathrm{PP}}_{\mathrm{OP}}$ quarter-hours. A typical value is 1 hour or $D^{\mathrm{PP}}_{\mathrm{OP}} = 4$. We call this constraint *minimum time operational level constraint*.

Let the binary variables χ^{C}_t keep track, if there is a change in the power plant state at the beginning of the time period t

$$\chi^{\mathrm{C}}_t \geq \delta_{mt} - \delta_{mt-1} \quad , \qquad \forall m, \quad t = 2, \ldots, N^{\mathrm{T}} \quad , \tag{14}$$

and

$$\chi^{\mathrm{C}}_t \geq \delta_{mt-1} - \delta_{mt} \quad , \qquad \forall m, \quad t = 2, \ldots, N^{\mathrm{T}} \quad . \tag{15}$$

Inequalities (14) and (15) ensure that variable χ^{C}_t has value 1, if there is a change in the state of the plant. This allows us to formulate the condition

$$\sum_{k=1}^{D^{\mathrm{PP}}_{\mathrm{OP}}} \chi^{\mathrm{C}}_{t+k-1} \leq 1 \quad , \qquad t = 1, \ldots, N^{\mathrm{T}} - \left(D^{\mathrm{PP}}_{\mathrm{OP}} - 1\right) \quad , \tag{16}$$

ensuring at most one state change within any $D^{\mathrm{PP}}_{\mathrm{OP}}$ time interval.

Similar to χ^{S}_t and χ^{I}_t, the binary variables χ^{C}_t can be relaxed to be non-negative continuous.

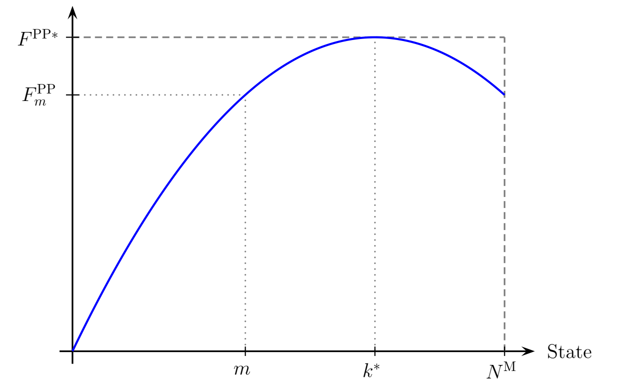

Fig. 1. Efficiency curve of a power plant over the continuum of operation and with respect to the discrete states

6) Minimum / Maximum Energy per Day: Typically, a power utility is given a lower and an upper bound for the energy production of its own power plant. Usually, these bounds are derived from mid-term or long-term optimization models [21], [22].

Let $E^{\mathrm{PP}}_{\mathrm{min}}$ ($E^{\mathrm{PP}}_{\mathrm{max}}$) be the minimum (maximum) energy to be generated by the utility's power plant. The constraint on the minimum / maximum energy production per day can then be modeled as

$$E^{\mathrm{PP}}_{\mathrm{min}} \leq \sum_{t=1}^{96} \sum_{m=1}^{N^{\mathrm{M}}} F_t L_m \delta_{mt} \leq E^{\mathrm{PP}}_{\mathrm{max}} \quad . \tag{17}$$

7) Initial Conditions: At the beginning of the day-ahead planning, the power plant is either in the idle state or operates at a particular state. This is captured in the model by fixing the appropriate variable δ_{m1} to value 1.

Furthermore, variables χ^{C}_t have to be fixed to 0 for some preceding time periods not allowing thus a change in the power plant operation due to the minimum time operational level restriction resulting from the previous day. Similarly, certain variables χ^{C}_t and χ^{I}_t have to be fixed to 0, dependent on the power plant's initial state.

G. Power Plant Cost

The operation of the power plant involves certain costs. The power plant has a variable generation cost per MWh, dependent on the efficiency level the plant is operated. CO2 certificates have to be bought in the market for each ton of CO2 produced by the power plant. In addition, when the power plant is started from the idle state, then start-up costs apply.

1) Operation Cost: The efficiency curve of the power plant is given through a curve of the type shown in Figure 1. We denote by F^{PP}_m the efficiency level of the power plant operated at state m ($\neq 1$); *i.e.*, m not being the idle state. F^{PP}_m is > 0 and is less than 1. The optimal operation with respect to efficiency is obtained at the level of k^* for the optimal efficiency of $F^{\mathrm{PP}*}$. However, recognize that k^* might not be one of the N^{M} states.

The cost in € for producing 1 MWh under the optimal cost efficient way is given by P^{PP}. Then, the variable production cost operating in state m is proportional to P^{PP} and inverse proportional to F_m^{PP}. Hence, the variable production cost in € per MWh is given by

$$\frac{1}{F_m^{\mathrm{PP}}} P^{\mathrm{PP}} \quad . \tag{18}$$

2) CO2 Emission Cost: We assume that our electric public utility is located in an environment with a 'cap and trade' mechanism similar to the European Union Emissions Trading Scheme (ETS) [23]. Hence, the CO2 emissions of the power plant can now be priced. Currently, for the ETS, the CO2 emission price in € per ton CO2 emissions is the same within the day.

Let us assume that the CO2 price is given by P^{CO2} and let F_m^{CO2} be the CO2 emissions in tons per MWh when the power plant is operated at state m. Then, the cost for the CO2 emissions is given by

$$P^{\mathrm{CO2}} \sum_{t=1}^{96} \sum_{m=2}^{N^{\mathrm{M}}} F_m^{\mathrm{CO2}} F_t L_m \delta_{mt} \quad . \tag{19}$$

Notice that the CO2 emission cost might be an opportunity cost for the utility, as the utility is issued a certain amount of CO2 allowances for free.

3) Start-up Cost: If the power plant starts its operation, certain costs apply. These costs are called start-up costs. They can also be seen as the fixed costs of the power plant operation.

If this fixed cost is $C_{\mathrm{SU}}^{\mathrm{PP}}$, then the start-up cost in € can be modeled as

$$C_{\mathrm{SU}}^{\mathrm{PP}} \sum_{t=1}^{96} \chi_t^I \quad , \tag{20}$$

where variable χ_t^I indicates if the power plant left the idle state at the beginning of period t, as introduced in Section III-F2.

4) Objective Function: The electric utility seeks to maximize the profit, given by the difference of the revenues and cost. The revenues are due to the sale of power in the power and balancing market and the costs are given by the operation of the power plant, including the CO2 emission costs. Hence, the objective can be summarized as

$$\begin{aligned} \max \quad & \sum_{h=1}^{24} \left(P_h^{\mathrm{PM}} - P_{\mathrm{T}}^{\mathrm{PM}}\right) F_h P_h^{\mathrm{PM,s}} + \sum_{t=1}^{96} P_t^{\mathrm{BM,s}} F_t p_t^{\mathrm{BM,s}} \\ & - \sum_{h=1}^{24} \left(P_h^{\mathrm{PM}} + P_{\mathrm{T}}^{\mathrm{PM}}\right) F_h p_h^{\mathrm{PM,b}} - \sum_{t=1}^{96} P_t^{\mathrm{BM,b}} F_t p_t^{\mathrm{BM,b}} \\ & - \sum_{t=1}^{96} P_t^{\mathrm{LF}} \left(p_t^{\mathrm{LF}}\right) - C_{\mathrm{SU}}^{\mathrm{PP}} \sum_{t=1}^{96} \chi_t^I \\ & - \sum_{t=1}^{96} \sum_{m=2}^{N^{\mathrm{M}}} \left(\frac{1}{F_m^{\mathrm{PP}}} C^{\mathrm{PP}} + F_m^{\mathrm{CO2}} P^{\mathrm{CO2}}\right) F_t L_m \delta_{mt} \quad , \end{aligned} \tag{21}$$

with the revenues from the spot market sales and balancing market sales and the cost resulting from the spot market purchases, the balancing market purchases, the purchases of electricity from the load following contract, the startup cost, the variable operation cost and the CO2 emission cost.

IV. Discussion

The modeling of the power plant operation, including all constraints, is implemented through a MILP formulation that has the favorable property to use binary variables only to model the states of the power plant per time interval. Hence, the number of binary variables of the model is given by $N^{\mathrm{M}} N^{\mathrm{T}}$; *i.e.*, for 8 states and 96 quarter-hour time intervals the model contains 768 binary variables.

Recognize that the purchases of electricity from the spot market involve non-negative integer variables. There are two integer variables per hour, one for purchases and one for sales, leading to 48 integer variables.

The integer variables corresponding to the spot market could be relaxed to be non-negative and the difference in MWh to an integral value can be made up for by the balancing market. However, with this method, there is no guarantee that the obtained solution is globally optimal.

When looking at the objective function and the constraints of the model, one can ask if the optimal solution is unbounded. Indeed, this is the case when there is an arbitrage opportunity; *i.e.*, if the electricity price from the balancing market is lower than the price from the spot market. However, when dealing with real market data, such behavior is not expected.

The model is very generic and can be extended in numerous ways. One could include base and peak load contracts, traded in the spot market. The modeling would involve two additional non-negative integer variables. The presence of the binary variables δ_{mt} allows also to include forced operation of the power plant at any given time at a certain state as well as forced shut down; *i.e.*, due to scheduled maintenance [2].

V. Conclusion

We presented an optimization model for an electric public utility in a liberalized market, optimizing the dispatch of its own power plant while trading in the market. The power plant has numerous constraints, limiting its operation, and a variety of costs associated with the power plant, such as operational cost, start-up cost and CO2 emission cost. The discussed model emphasizes on the dispatch of the power plant in discrete steps.

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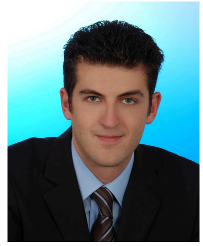

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