Stochastic Hydro-Thermal Scheduling under CO_2 Emissions Constraints

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***Abstract*—Despite the uncertainty surrounding the design of a mechanism which is ultimately accepted by nations worldwide, the necessity to implement regulations to curb emissions of greenhouse gases on a global scale is consensual. The electricity sector plays a fundamental role in this puzzle and countries may soon have to revise their operating policy directives in order to make them compatible with additional constraints imposed by such regulations. We present a modeling approach for greenhouse gas emissions quotas which can be incorporated into a stochastic dual dynamic programming algorithm, commonly used to solve the hydro-thermal scheduling problem. Our approach is flexible and capable of accommodating a detailed representation of emissions and related constraints. A case study based on the Guatemalan power system exemplifies the potential effects of considering these restrictions.**

***Index Terms*—CO_2 emissions, hydro-thermal scheduling, stochastic programming, stochastic dual dynamic programming**

I. Nomenclature

Sets:

$\mathbf{I}$	hydro plants / reservoirs, $i \in \mathbf{I} = \{1, \ldots, I\}$
$\mathbf{J}$	thermal plants, $j \in \mathbf{J} = \{1, \ldots, J\}$
$\mathbf{L}$	backward openings, $l \in \mathbf{L} = \{1, \ldots, L\}$
$\mathbf{M}$	forward inflow scenarios, $m \in \mathbf{M} = \{1, \ldots, M\}$
$\mathbf{N}$	linear segments of the future cost function underestimator, $n \in \mathbf{N} = \{1, \ldots, N\}$
Ω_t	random outcomes (hydro inflows) conditioned on previous outcomes, $\omega_t \in \Omega_t$
$\mathbf{R}$	CO_2 emissions reservoirs, $r \in \mathbf{R} = \{1, \ldots, R\}$
$\mathbf{T}$	time stages of the model, *i.e.*, $T = 12$ for a 1 year model with monthly stages, $t \in \mathbf{T} = \{1, \ldots, T\}$
$\mathbf{T}^1$	time stages except the first stage, *i.e.*, $\mathbf{T} \setminus \{1\}$
$\mathbf{U}_i$	hydro plants directly upstream of hydro plant i
$\mathbf{Y}^g$	stages when the CO_2 emissions allowances are given, $\mathbf{Y}^g = \{1, \ldots, Y^g\} \subseteq \mathbf{T}$
$\mathbf{Y}^e$	stages when the CO_2 emissions allowances expired, $\mathbf{Y}^e = \{1, \ldots, Y^e\} \subseteq \mathbf{Y}^g$

Input Data:

a_0	hydro inflows at the stage(s) prior to the start of the planning horizon as a vector of the hydro reservoirs, [m^3]
a_t	hydro inflow for stage t as a vector of the hydro reservoirs, may depend on i, l, and ω_t, [m^3]
B_j	(average) CO_2 emissions factor for each thermal plant j, [tons CO_2/MWh]
c_{jt}	variable electricity generation cost for thermal plant j for stage t, [\$/MWh]
C^{CO2}	fine for exceeding the CO_2 emissions allowances, [\$/ton CO_2]
d_t	electricity demand for stage t, [MWh]
e_0	initial CO_2 emissions allowances, [tons CO_2]
$E_y^{\text{CO}_2}$	CO_2 emissions quota per horizon, [tons CO_2]
$\underline{g}_{jt}, \bar{g}_{jt}$	lower/upper bound on power generation for thermal plant j and stage t, [MWh]
$\gamma_{tn}^{\{v,e,a\}}$	slope of future cost function underestimator for hydro reservoir level (v), emissions reservoir level (e) or past hydro inflow (a) for stage t and linear segment n
γ_{tn}^c	constant of future cost function underestimator for stage t and linear segment n
p_l	conditional probability of water inflow scenario l
π_t^{ln}	dual multipliers of CO_2 emissions reservoir constraints for stage t, water inflow scenario l and linear segment n
ρ_i	power generation coefficient for hydro plant i, [MWh/m^3]
$\underline{s}_{it}, \bar{s}_{it}$	lower/upper bound on spillage for hydro reservoir i and stage t, [m^3]
$\underline{u}_{it}, \bar{u}_{it}$	lower/upper bound on turbined water for hydro reservoir i and stage t, [m^3]
Υ	fine for electricity demand rationing, [\$/MWh]
$\underline{v}_{it}, \bar{v}_{it}$	lower/upper bound on reservoir level for hydro reservoir i and stage t, [m^3]
v_1	initial hydro reservoir level, may depend on i, [m^3]

Functions:

z	hydro-thermal operation cost
z_t^l	cost function for stage t and inflow scenario l
z_{t+1}	future cost function for stage t

Variables:

δ_t	demand rationing for stage t, may depend on l or ω_t, [MWh]
$\mathbf{e}_{t+1}$	CO_2 emissions reservoir level at the end of stage t, may depend on l, ω_t or r, [tons CO_2]
$\mathbf{f}_t$	CO_2 emissions above quota for stage t, may depend

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on l, ω_t or r, [tons CO_2]

$\mathbf{g}_{jt}$ electricity generation for thermal plant j and stage t, may depend on l or ω_t, [MWh]

$\mathbf{s}_{it}$ spillage for hydro reservoir i and stage t, may depend on l or ω_t, [m^3]

$\mathbf{u}_{it}$ turbined water by hydro plant i for stage t, may depend on l or ω_t, [m^3]

$\mathbf{v}_{it+1}$ hydro reservoir level at the end of stage t and the beginning of stage $t+1$ for hydro reservoir i, may depend on l or ω_t, [m^3]

$\mathbf{x}_t$ CO_2 emitted at stage t, [tons CO_2]

II. Introduction

We study the mid-term stochastic optimization problem of a hydro-thermal power system which is subject to CO_2 emissions quotas. This problem is important and timely as world leaders and international organizations discuss the roles and responsibilities of each country and sector of economic activity in the path towards a sustainable future.

When deciding on the imposition of emissions quotas on a power system, one of the main concerns for each country is the impact of such limitations on the competitiveness of its industrial activities and the potential side effects on its economy. The existence of a quota and penalties associated with its violation may have huge effects in terms of decreasing economic activity and additional costs related to energy efficiency projects, higher energy costs and eventual acquisition of additional quotas in international markets.

Since the emissions quotas are to be established for each country as a whole, it is up to each government to decide how is is going to be divided among each sector – and this is where the problem we study in this paper is relevant. Having a limit on the total CO_2 emissions allowances over a fixed horizon (say, one year) directly affects the way centralized system operators – such as the ones existing in countries in South and Central America – define the operating schedule of each plant because a new element must be considered in addition to the usual sources of uncertainty such as demand and inflows. While it is desirable that the generation of dirty plants is replaced by that of cleaner alternatives, this comes at a cost which must be borne by society. It thus becomes imperative that policy makers are able to estimate the increase in costs when defining the share of quotas to be allocated to the electricity sector and the fines associated with their violations.

Managing an annual emissions allowance is somewhat similar to managing water reservoirs since one must determine the optimal tradeoff between consuming parts of the limited amount of a resource in the present moment or saving it for future use. The decision to deplete the CO_2 stock on hand may only be assessed in terms of its expected future costs which depend on the evolution of hydrological conditions. For example, consuming emissions quotas in the present – thus preventing their use in future time stages – may prove useful if a high inflow scenario occurs and hydro plants are able to meet a higher share of demand.

Belsnes et al. [1] model CO_2 emissions reservoirs in SDDP via a "hydro" reservoir, where thermal plants are interpreted as hydro-electric stations using CO_2 allowances (instead of water) and transmission line costs are used to model the thermal generation costs. We introduce an alternative reservoir model, allowing complicating constraints (*e.g.*, fuel availability) on the thermal plants and allowing CO_2 emissions to expire as determined by the EU Emission Trading Scheme regulations.

The contribution of this work is on the modeling aspect of the emissions-constrained hydro-thermal scheduling problem. We propose a representation of greenhouse gas emissions quotas as reservoirs, thus allowing it to be readily embedded into a stochastic dual dynamic programming (SDDP) algorithm with the addition of state variables into the cost-to-go functions. The proposed approach is flexible and capable of handling the representation of emissions constraints both at a system-wide level and at a more detailed level that encompasses different quotas for each technology or set of plants. While there are numerous economic assessments and policy analyses stemming from the results of the problem being studied, that is not the focus of this work.

The remainder of the paper is organized as follows. In Section III, we introduce the problem of interest and review the solution methods. Our CO_2 emissions reservoir model appears in Section IV along with the derivation of the future function cuts. A case study based on the Guatemalan power system is presented in Section V. We conclude with Section VI.

III. Stochastic Hydro-Thermal Programming with CO_2 Emissions Reservoirs

In this section, we describe the hydro-thermal scheduling problem with CO_2 emissions constraints.

Given is a hydro-thermal system with I hydro power plants $i \in \mathbf{I} = \{1, \dots, I\}$ and J thermal plants $j \in \mathbf{J} = \{j, \dots, J\}$. Decisions can be made at discrete stages $t \in \mathbf{T} = \{1, \dots, T\}$, *e.g.*, monthly, on the electricity generation $\mathbf{g}_{jt}$ of the thermal plants $j \in \mathbf{J}$ and the electricity generation $\rho_i u_{it}$ of the hydro power plants $i \in \mathbf{I}$.

The objective is the minimization of the expected operational cost z of the system over the whole horizon, particularly taking into account CO_2 emissions quotas. The operation cost consists of the variable cost for the electricity production of the thermal plants and the fees to be paid if the emissions quotas are exceeded. The variable production costs of hydro plants are assumed to be zero.

CO_2 is emitted at a constant factor B_j [tons CO_2/MWh] whenever the thermal plant $j \in \mathbf{J}$ is used to generate electricity. To avoid short term anomalies (*e.g.*, due to extreme weather), CO_2 emissions allowances are not given at every stage (*e.g.*, monthly), but for a longer time period (*e.g.*, yearly), *i.e.*, at stages $y \in \mathbf{Y}^g \subseteq \mathbf{T}$. If total emissions exceed the given quota E_y^{CO2}, then a penalty fee of C^{CO2} [\$/ton CO_2] for the emissions exceeding the quota has to be paid. We distinguish two models where the emissions allowances expired at a certain term $y \in \mathbf{Y}^e$ and where the emissions allowances have no expiration date once they are issued.

Without loss of generality, we assume that each hydro plant has a water reservoir and vise versa. We will not distinguish between reservoirs and hydro plants except when explicitly specified and we give both the index $i \in \mathbf{I}$. The inflows a_{it} [m^3] per hydro reservoir $i \in \mathbf{I}$ and stage $t \in \mathbf{T}$ are assumed to be stochastic while all other technical specifications of the system are known; in particular, the generation cost c_{jt} [\$/MWh] per thermal plant $j \in \mathbf{J}$ and stage $t \in \mathbf{T}$, as well as the electricity demand d_t [MWh] per stage $t \in \mathbf{T}$, are given as an average value over the time discretization, *i.e.*, monthly. These assumptions are not particularly necessary for the modeling approach being studied in this paper. However, for ease of presentation and interpretation of results, we opted to focus on the CO_2 emissions reservoir modeling. We formalize the problem by casting it as a mathematical program.

A. Stochastic Programming Model

Let Ω_t be the set of all random outcomes conditioned on the previous stages and ω_t be a realization. Each random outcome reveals a certain hydro inflow scenario for the corresponding stage. Then, the described mid-term hydro-thermal scheduling problem can be modeled as the following multi-stage stochastic linear programming problem

$$z := \min \sum_{j \in \mathbf{J}} c_{j1}\mathbf{g}_{j1} + \Upsilon\delta_1 + C^{\mathrm{CO2}}\mathbf{f}_1 + \\ + \min \mathbb{E}_{\omega_2 \in \Omega_2}\left[\sum_{j \in \mathbf{J}} c_{2j}\mathbf{g}_{2j}(\omega_2) + \Upsilon\delta_2(\omega_2) + C^{\mathrm{CO2}}\mathbf{f}_2(\omega_2) + \ldots + \\ + \min \mathbb{E}_{\omega_T \in \Omega_T}\left[\sum_{j \in \mathbf{J}} c_{Tj}\mathbf{g}_{Tj}(\omega_T) + \Upsilon\delta_T(\omega_T) + C^{\mathrm{CO2}}\mathbf{f}_T(\omega_T)\right]\ldots\right] \quad (1)$$

$$\text{s.t.} \quad \sum_{j \in \mathbf{J}} \mathbf{g}_{j1} + \sum_{i \in \mathbf{I}} \rho\mathbf{u}_{i1} + \delta_1 = d_1 \quad (2)$$

$$\sum_{j \in \mathbf{J}} \mathbf{g}_{jt}(\omega_t) + \sum_{i \in \mathbf{I}} \rho\mathbf{u}_{it}(\omega_t) + \delta_t(\omega_t) = d_t, \quad t \in \mathbf{T}^1 \quad (3)$$

$$\mathbf{v}_{i2} = v_{i1} - \mathbf{u}_{i1} - \mathbf{s}_{i1} + \sum_{h \in \mathbf{U}_i} (\mathbf{u}_{h1} + \mathbf{s}_{h1}) + a_{i1}, \quad i \in \mathbf{I} \quad (4)$$

$$\mathbf{v}_{it+1}(\omega_t) = \mathbf{v}_{it}(\omega_{t-1}) - \mathbf{u}_{it}(\omega_t) - \mathbf{s}_{it}(\omega_t) + \\ + \sum_{h \in \mathbf{U}_i} (\mathbf{u}_{ht}(\omega_t) + \mathbf{s}_{ht}(\omega_t)) + a_{it}(\omega_t), \quad t \in \mathbf{T}^1, i \in \mathbf{I} \quad (5)$$

$$\sum_{t|y} \left(\sum_{j \in \mathbf{J}} B_j\mathbf{g}_{jt}(\omega_t) - \mathbf{f}_t(\omega_t)\right) \le E_y^{\mathrm{CO2}}, \quad y \in \mathbf{Y} \quad (6)$$

$$\underline{g}_{j1} \le \mathbf{g}_{j1} \le \bar{g}_{j1}, \qquad \underline{g}_{jt} \le \mathbf{g}_{jt}(\omega_t) \le \bar{g}_{jt}, \\ \underline{u}_{i1} \le \mathbf{u}_{i1} \le \bar{u}_{i1}, \qquad \underline{u}_{it} \le \mathbf{u}_{it}(\omega_t) \le \bar{u}_{it}, \\ \underline{v}_{i2} \le \mathbf{v}_{i2} \le \bar{v}_{i2}, \qquad \underline{v}_{it+1} \le \mathbf{v}_{it+1}(\omega_t) \le \bar{v}_{it+1}, \\ \underline{s}_{i1} \le \mathbf{s}_{i1} \le \bar{s}_{i1}, \qquad \underline{s}_{it} \le \mathbf{s}_{it}(\omega_t) \le \bar{s}_{it}, \\ \mathbf{f}_1 \ge 0, \qquad \mathbf{f}_t(\omega_t) \ge 0, \\ \delta_1 \ge 0, \qquad \delta_t(\omega_t) \ge 0, \quad t \in \mathbf{T}^1, i \in \mathbf{I}, j \in \mathbf{J}. \quad (7)$$

The hydro inflows a_{i1} are known with certainty in the first stage when the generation decisions are made while the inflows and the decisions are stochastic in all other stages.

Let us now examine the future stages. There, the electricity demand d_t has to be met by thermal and/or hydro electricity production as stated in constraints (3) where a rationing of $\delta_t(\omega_t)$ is possible but penalized in the objective function. The water balance equations (5) ensure that the reservoir level $\mathbf{v}_{it+1}(\omega_t)$ for reservoir i at the end of stage t equals the reservoir level $\mathbf{v}_{it}(\omega_{t-1})$ at the beginning of the stage plus the stochastic water inflow $a_{it}(\omega_t)$ minus the turbined water $u_{it}(\omega_t)$ minus the spilled water s_{it} plus the inflows from the plants immediately upstream of plant i, either from upstream hydro power generation or spillage, with the notation $\mathbf{v}_{i2}(\omega_1) \equiv \mathbf{v}_{i2}$.

Constraints (6) model the emissions allowances over the horizon, where the emitted tons of CO_2 due to thermal generation must be less than or equal to the sum of the CO_2 quota E_y^{CO2} and the additional CO_2 allowances "bought" via fines $\mathbf{f}_t$. The notation $t|y$ indicates whether or not stage t corresponds to the emissions allowance period starting at y and ending at the period before $y+1$, *i.e.*, $x := y+1 \in \mathbf{Y}$ and $t \in \mathbf{T}$ with $t+1 = x$. This constraint may span multiple stages which is problematic for the decomposition methods available. We discuss this in greater detail in Section IV.

The objective function (1) is then given as the sum of the first stage generation cost plus the expected cost of thermal power generation in the future stages, including possible fees for rationing and CO_2 emissions quota violations.

At this point, it is worth mentioning that model (1)-(7) can be slightly modified to reflect the possibility of selling allowances in excess of usage and thus fully capturing the so-called *cap-and-trade* schemes. This can be accomplished by replacing variables $\mathbf{f}_t$ by the difference of two non-negative variables, where $\mathbf{f}_t$ can then be interpreted as the net purchase of CO_2 emissions at stage $t \in \mathbf{T}$. The CO_2 emissions fine C^{CO2} could then be replaced by the associated cost and revenue resulting from this trade, as an estimation of the quota prices in international markets. We have, however, adopted the role of a central planner or policy maker whose primary objective is not to profit from the possibility of quota trading but rather to define consistent and achievable emissions goals.

Additional linear operational constraints can be added to the model (1)-(7) to make it more practical, *e.g.*, linearized electricity and gas network constraints, multiple load blocks, and sub-systems. A detailed discussion of these operational constraints can be found in [2]–[4].

B. Solution Methods

There are different methods in the literature on how to solve problem (1)-(7) for the case in which the CO_2 emissions constraints (6) are not present. We focus our discussion on sampling based methods in a dynamic programming framework, which sample the hydro inflows and solve a series of linear programs using a stage decomposition.

A review on hydro-thermal scheduling algorithms until the beginning of the 1980s was presented in [5]. However, a remarkable series of research followed based on nested Benders Decomposition methods. Among them are the papers by Pereira and Pinto [2], Jacobs et al. [6] and Morton [7]. In 1991, the stochastic dual dynamic programming algorithm developed by Pereira and Pinto [8] was proposed with the aim of overcoming the biggest drawback of the previously used methods such as stochastic dynamic programming: the curse of dimensionality.

Since it was originally proposed, SDDP has been analyzed and extended. The mathematical foundations of SDDP have also been studied by a few authors who have derived interesting results regarding its statistical and convergence properties. Among these, Gjelsvik et al. [9] review SDDP methods which have been applied to the hydro-dominated Nordic countries, with a special focus on their developed hybrid method combining stochastic dynamic programming with SDDP [10]. A convergence analysis for sampling-based methods was given by Philpott and Guan [11] while results on the convergence speed were obtained by Linderoth et al. [12] for two-stage stochastic linear programs with recourse.

A similar technique to SDDP called constructive dual dynamic programming (CDDP) was developed by Read [13] in 1984. The basic idea is to solve the dual of the dynamic programming formulation directly. This allows the construction of the marginal value surface *exactly*, defining an operating policy over the whole state-space, *cf.*, [14].

Assuming the absence of constraints (6), the basic idea of SDDP is to decompose the optimization problem at hand into separate problems for each stage, where the stages are connected via future cost functions expressed in terms of state variables. The algorithm is essentially composed of two phases which are performed at each iteration. The backwards pass derives its name from the fact that during its execution the algorithm solves one-stage problems in reverse chronological order. The solution of a problem at stage t is used to generate a cut – *i.e.*, a supporting hyperplane – providing an approximation to future costs associated with decisions taken at stage $t-1$. Once the algorithm has progressed to the first stage, a Monte Carlo forward simulation is carried out by solving one-stage problems which take into account the previously calculated cuts – in this case the problem of stage t is connected to that of stage $t+1$ via initial storage values. A lower bound on the objective function value is obtained from the solution of the first-stage problem of the backward pass, which may then be compared against the upper bound provided by the sum of the costs along the horizon given by the problems solved during the forward simulation.

At each stage t of the backwards pass, L different inflow scenarios a^l_{it}, $l \in \mathbf{L} = \{1, \ldots, L\}$ are assumed for each forward inflow scenario m, the so-called "backwards openings;" for simplicity, we omit index $m \in \mathbf{M}$ in subsequent discussions. The lateral inflows are modeled as a multivariate periodic autoregressive model with lag p, capturing the autocorrelation of the inflows at stage t to the inflow of the previous p stages. This way, the expectation in the objective function for each stage decomposes into a weighted sum of the inflow scenarios. Furthermore, as the L inflows per stage are assumed to be independent from each other, each one-stage dispatch problem separates into L independent problems. For the following discussions, we assume without loss of generality an inflow model with lag 1.

IV. CO_2 Emissions Allowances Modeling via Reservoirs

Constraint (6) may span multiple stages and, hence, destroys the block diagonal structure of the constraint matrix. Decomposition methods exploit this structure and SDDP cannot handle constraint (6) in its present form. Hence, we suggest a formulation of the CO_2 emissions quotas respecting the stage decomposition framework of SDDP.

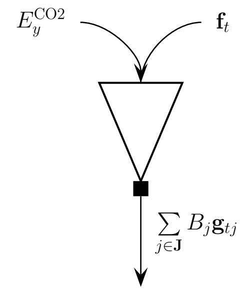
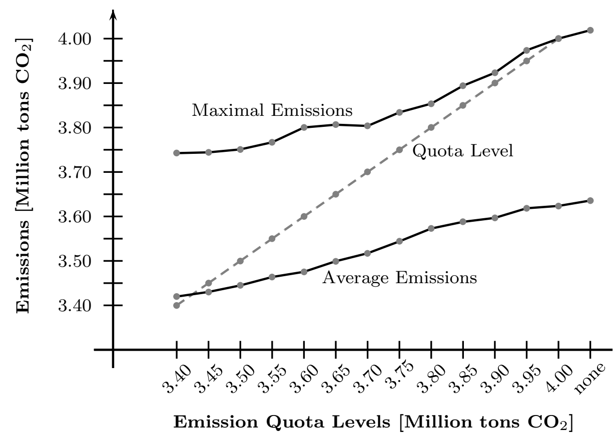
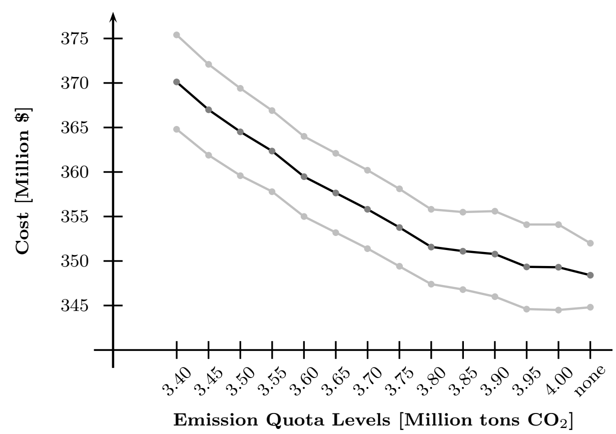


Fig. 1. CO_2 emissions reservoirs



(a) Average and maximum CO_2 emissions



(b) Average operational cost with 95% confidence interval

Fig. 2. CO_2 emissions and operational cost for different quota levels

The CO_2 emissions allowances, modeled via constraint (6), can be interpreted as a reservoir as follows: At any given time $y \in \mathbf{Y}^g$, *e.g.*, at the beginning of the year, it "rains" CO_2 emissions rights, filling the emissions reservoir; see Fig. 1. At each time stage $t \in \mathbf{T}$, there is a balance equation for the CO_2 emissions as follows

$$\mathbf{e}_{t+1} = \mathbf{e}_t - \sum\nolimits_{j\in\mathbf{J}} B_j \mathbf{g}_{jt} + \mathbf{f}_t, \quad t \in \mathbf{T} \setminus \mathbf{Y}^g \tag{8}$$

$$\mathbf{e}_{t+1} = \widetilde{\mathbf{e}}_t - \sum\nolimits_{j\in\mathbf{J}} B_j \mathbf{g}_{jt} + \mathbf{f}_t + E_t^{CO2}, \quad t \in \mathbf{Y}^g \tag{9}$$

$$\mathbf{e}_{t+1} \geq 0, \quad \mathbf{f}_t \geq 0, \quad t \in \mathbf{T}, \tag{10}$$

with the initial emissions allowances e_1 at the beginning of the planning period. In equations (8), the emissions allowances remaining at the end of stage t, $\mathbf{e}_{t+1}$, are the CO_2 emissions allowances at the beginning of stage t, $\mathbf{e}_t$, minus the emissions allowances used via thermal electricity generation, $\sum_{j\in\mathbf{J}} B_j \mathbf{g}_{jt}$, plus the emissions rights fined, $\mathbf{f}_t$, plus the emissions allowances issued, E_t^{CO2}. We require a (non-negative) variable $\mathbf{f}_t$ for the emissions exceeding the quota for each stage $t \in \mathbf{T}$ (in contrast to $y \in \mathbf{Y}^g$) to ensure that the emissions reservoir level $\mathbf{e}_{t+1}$ is non-negative at the end of each stage.

With equation (9), we are able to model two cases: (i) the emissions allowances expire at (the beginning of) stages $y \in \mathbf{Y}^e$ or (ii) there is no expiration date. This is realized via definition

$$\widetilde{\mathbf{e}}_t := \begin{cases} 0, & \text{if the allowances expire, } i.e.,\ t \in \mathbf{Y}^e \\ \mathbf{e}_t, & \text{if the allowances do not expire} \end{cases} \tag{11}$$

for all $t \in \mathbf{T}$, where $\mathbf{e}_1$ represents the initial CO_2 emissions allowances carried over from previous time periods. This model permits emissions allowances to expire when new allowances are issued, as is the case in the EU Emission Trading Scheme, *i.e.*, $\mathbf{Y}^e \subseteq \mathbf{Y}^g$.

The proposed approach is also able to handle allowances lasting longer than the timespan between the moments when they are issued. This could be accomplished by adding as many CO_2 emissions reservoirs as there are simultaneously valid allowances. Therefore, let $r \in \mathbf{R} = \{1, \dots, R\}$ be the set of CO_2 emissions reservoirs needed. Equations (8)-(10) can then be replaced by

$$\mathbf{e}_{t+1}^r = \mathbf{e}_t^r - \mathbf{x}_t^r + \mathbf{f}_t^r, \quad t \in \mathbf{T} \setminus \mathbf{Y}^g, r \in \mathbf{R} \tag{12}$$
$$\mathbf{e}_{t+1}^r = \widetilde{\mathbf{e}}_t^r - \mathbf{x}_t^r + \mathbf{f}_t^r + E_t^{\mathrm{CO2}}, \quad t \in \mathbf{Y}^g, r \in \mathbf{R} \tag{13}$$
$$\mathbf{e}_{t+1}^r \geq 0, \quad \mathbf{f}_t^r \geq 0, \quad \mathbf{x}_t^r \geq 0, \quad t \in \mathbf{T}, r \in \mathbf{R}, \tag{14}$$

with

$$\sum\nolimits_{j\in\mathbf{J}} B_j \mathbf{g}_{jt} = \sum\nolimits_{r\in\mathbf{R}} \mathbf{x}_t^r \quad \text{and} \quad \mathbf{f}_t = \sum\nolimits_{r\in\mathbf{R}} \mathbf{f}_t^r, \quad t \in \mathbf{T},$$

where $\widetilde{\mathbf{e}}_t^r$ indicates whether or not emissions in the CO_2 emissions reservoir $r \in \mathbf{R}$ expire at $t \in \mathbf{Y}^e$.

A. One-Stage Dispatch Problem

Modeling the CO_2 emissions quotas as reservoirs through constraints (8)-(10) allows a stage decomposition of the hydro-thermal scheduling problem. At stage t, coupling previous and future stages is then given by the following so-called "state variables:" the hydro reservoir levels v_t as the "initial" reservoir level for stage t, the CO_2 emissions reservoir level e_t, and the previous inflows a_{t-1} – the inflow forecasts a_{it}^l for stage t depend linearly on a_{it-1} (*cf.* Section III-B). Decomposing the problem into stages and inflow scenarios $l \in \mathbf{L}$ each with (conditional) probability, the following one-stage dispatch problem is obtained

$$z_t^l(v_t, e_t, a_{t-1}) = \min \sum_{j\in\mathbf{J}} c_{jt} \mathbf{g}_{jt}^l + \Upsilon \delta_t^l + C^{\mathrm{CO2}} \mathbf{f}_t^l + \\ + z_{t+1}\left(\mathbf{v}_{t+1}^l, \mathbf{e}_{t+1}^l, a_t^l\right) \tag{15}$$

$$\text{s.t.} \sum\nolimits_{j\in\mathbf{J}} \mathbf{g}_{jt}^l + \sum\nolimits_{i\in\mathbf{I}} \rho_i \mathbf{u}_{it}^l + \delta_t^l = d_t \tag{16}$$
$$\mathbf{v}_{it+1}^l = v_{it} - \mathbf{u}_{it}^l - \mathbf{s}_{it}^l + \sum\nolimits_{h\in U_i} \left(\mathbf{u}_{ht}^l + \mathbf{s}_{ht}^l\right) + a_{it}^l, \quad i \in \mathbf{I} \tag{17}$$
$$\mathbf{e}_{t+1}^l = e_t - \sum\nolimits_{j\in\mathbf{J}} B_j \mathbf{g}_{jt}^l + \mathbf{f}_t^l, \quad t \in \mathbf{T} \setminus \mathbf{Y}^g \tag{18}$$
$$\mathbf{e}_{t+1}^l = \widetilde{\mathbf{e}}_t - \sum\nolimits_{j\in\mathbf{J}} B_j \mathbf{g}_{jt}^l + \mathbf{f}_t^l + E_t^{\mathrm{CO2}}, \quad t \in \mathbf{Y}^g \tag{19}$$
$$\underline{g}_{jt} \leq \mathbf{g}_{jt}^l \leq \bar{g}_{jt}, \quad \underline{u}_{it} \leq \mathbf{u}_{it}^l \leq \bar{u}_{it},$$
$$\underline{v}_{it+1} \leq \mathbf{v}_{it+1}^l \leq \bar{v}_{it+1}, \quad \underline{s}_{it} \leq \mathbf{s}_{it}^l \leq \bar{s}_{it},$$
$$\mathbf{e}_{t+1}^l \geq 0, \quad \mathbf{f}_t^l \geq 0, \quad \delta_t^l \geq 0, \quad j \in \mathbf{J}, i \in \mathbf{I}. \tag{20}$$

The objective function value z is then approximated through $z_1(v_1, e_1, a_0)$.

B. Future Function Cuts for CO_2 Emissions Reservoirs in SDDP

Given a mechanism to calculate or estimate the future cost functions z_{t+1} in (15), the one-stage dispatch problems and the original hydro-thermal scheduling problem (1)-(7) can be solved.

Therefore, SDDP uses the property that the future cost function $z_t(v_t, e_t, a_{t-1})$ is convex, jointly in its state variables v_t, e_t and a_{t-1}. The main reasons for its convexity are that the inflow model used is linear in the previous inflows and all three state variables appear as right hand sides in (17)-(19). Evaluating this function z_t at a specific point v_t^n, e_t^n and a_{t-1}^n leads to a function value $z_t(v_t^n, e_t^n, a_{t-1}^n) \in \mathbb{R}$. If we also know the slopes γ_{tn}^v, γ_{tn}^e and γ_{tn}^a of z_t at this point v_t^n, e_t^n and a_{t-1}^n, then we can extrapolate the function z_t. In other words, we can *underestimate* the function z_t via the (linear) slopes of the points v_t^n, e_t^n and a_{t-1}^n. Hence, we obtain a linear program, defining a lower bound on the "true" function z_t

$$\underline{z}_t = \min \alpha \tag{21}$$
$$\text{s.t. } \alpha \geq \gamma_{tn}^v v_t^n + \gamma_{tn}^e e_t^n + \gamma_{tn}^a a_{t-1}^n + \gamma_{tn}^c, \qquad n \in \mathbf{N} \tag{22}$$

where $n \in \mathbf{N} = \{1, \dots, N\}$ denotes the n-th linear segment of the convex underestimation and γ_{tn}^c is the corresponding constant term.

The slopes γ_{tn}^v and γ_{tn}^a are determined through the dual multipliers of the water balance equations (17), [3]. Similarly, γ_{tn}^e can be obtained via

$$\gamma_{tn}^e = \left.\frac{\partial z_t(\cdot, e_t, \cdot)}{\partial e_t}\right|_{e_t = e_t^n} = \left.\frac{\partial \left(\sum_{l\in\mathbf{L}} p_l z_t^l(\cdot, e_t, \cdot)\right)}{\partial e_t}\right|_{e_t = e_t^n} \tag{23}$$
$$= \sum_{l\in\mathbf{L}} p_l \left.\frac{\partial z_t^l(\cdot, e_t, \cdot)}{\partial e_t}\right|_{e_t = e_t^{ln}} \tag{24}$$

with probability p_l of inflow scenario l conditioned on the previous inflows. Now, let π_t^{ln} be the dual multipliers of constraint (18) for $t \in \mathbf{T} \setminus \mathbf{Y}^g$ and the dual multipliers of constraint (19) for $t \in \mathbf{Y}^g$, respectively, for a given emissions reservoir level e_t^n. Then, we obtain

$$\gamma_{tn}^e = \sum_{l\in\mathbf{L}} p_l \pi_t^{ln}, \quad t \in \mathbf{T} \setminus \mathbf{Y}^g, n \in \mathbf{N} \tag{25}$$

and for $y \in \mathbf{Y}^g$ and $n \in \mathbf{N}$

$$\gamma_{yn}^e = \begin{cases} 0, & \text{if the allowances expire, } i.e.,\ t \in \mathbf{Y}^e \\ \sum_{l\in\mathbf{L}} p_l \pi_y^{ln}, & \text{if the allowances do not expire} \end{cases}. \tag{26}$$

TABLE II
CO_2 EMISSIONS FACTORS USED FOR DIFFERENT TYPES OF THERMAL PLANTS. ALL OTHER PLANTS ARE ASSUMED TO HAVE ZERO EMISSIONS

Type	**CO_2 factor**	**Source**	**CO_2 factor** [kg CO_2 / MMBtu]
coal	2.86 [tons CO_2 / ton]	[15]	126.10
diesel	22.38 [pounds CO_2 / gallon]	[16]	72.66
bunker	78.80 [kg CO_2 / MMBtu]	[16]	78.80

The constant term γ^c_{tn} is then calculated from

$$\gamma^c_{tn} = \gamma^v_{tn} v^n_t + \gamma^e_{tn} e^n_t + \gamma^a_{tn} a_{t-1} - z_t(v^n_t, e^n_t, a^n_{t-1}). \quad (27)$$

The running time of each main iteration (*i.e.*, backwards and forward pass) of the SDDP algorithm with CO_2 emissions reservoirs is dominated by the number of linear programs to be solved, which is $\Theta(T \cdot M \cdot L \cdot N)$. Thus, the running time depends linearly on the parameters T, M, L and N, though it is not a priori clear what effects parameter changes have on the number of main iterations of SDDP algorithms. In practice, however, the number of main iterations tends to increase with an increase of the parameters T and/or M while increasing L and/or N tends to reduce the number of main iterations.

V. CASE STUDY FOR GUATEMALA

For our computational studies, we used a configuration based on the Guatemalan power system (disregarding, for simplicity, international interconnections, for example) as of the year 2008. The electricity power system consists of 41 thermal plants (Table I). The assumption of a constant fuel price throughout the planning horizon leads to a constant production price for each plant. Table II gives the CO_2 emissions factors per type of plant used to calculate the CO_2 emissions per plant of the Guatemalan power system.

Guatemala has one hydro-reservoir plant "Chixoy" with a hydro storage capacity of 440 hm^3 and ten run-of-the-river plants. Table III summarizes for each hydro-electric plant the installed capacities and the mean (∅), minimum (min.), maximum (max.) and standard deviation (std.) of the annual stochastic hydro inflows with a monthly and annual resolution.

The SDDP algorithm with the CO_2 emissions reservoir constraints has been implemented using the modeling language Mosel (Version 3.0.0) [17]. The resulting linear programs from the decomposition are solved using Xpress Optimizer (Version 20.00.05) [18]. Given the modeling language environment, this SDDP code was optimized for ease of implementation and readability, rather than computational efficiency. Nevertheless, all instances solved in between 15 and 50 minutes on a standard, single core laptop computer. The time horizon is one year with monthly stages where the first month is January and the last month is December. We apply an annual discount rate of 10%. Inflow uncertainty is captured within SDDP via scenarios which are generated by linear autoregressive models with correlated innovations – the estimation of parameters was carried out by fitting a lag-1 model to real inflow data from the past 38 years. We use 100 inflow scenarios for our simulations during the SDDP forward phase while 25 inflow scenarios are used for the backward pass. Hence, the presented results are "averages" over those 100 forward inflow scenarios.

The electricity demand for the planning horizon of one year is assumed to be known and follows a seasonal pattern (Table IV). The emissions fine for exceeding a given annual quota was set to $100 and the penalty for rationing was fixed to 10 times the largest generation cost.

Operating the Guatemalan power system without a CO_2 emissions quota leads to (average) annual operational costs of $348 million and (average) emissions of 3.635 million tons of CO_2. We impose different quota levels on the system (Fig. 2(a)). We observe that the average CO_2 emissions decrease at a slower rate than the quota levels decrease. Fig. 2(a) also shows the CO_2 emissions of the worst case scenario, *i.e.*, the case with the lowest inflows observed. Interestingly, the quota level of $100 does not push the worst case emissions below the quota level. Hence, the probability of exceeding the quota limit is above zero for all tested quota levels less than 3.95; the values are shown in Table V in row 4. Given an acceptable risk of exceeding the yearly allowance, one could easily identify the "optimal" penalty level by running the model several times.

Table V shows the average annual operation cost for different quota levels in row 2, including the incurred fines, while row 3 provides the corresponding half-width of the estimation. The average amount of CO_2 emissions exceeding the given quota is listed in row 4, while row 5 provides the number of scenarios exceeding the quota. Rows 6 and 7 report on the risk of not meeting the electricity demand. The numbers suggest that the rationing risk is negligible.

The average annual operation costs, together with the 95% confidence interval over all 100 scenarios, are shown in Fig. 2(b). The slope of the line can be interpreted as the incremental (operational) cost for CO_2 reduction, subject to some noise. When excluding CO_2 emissions fines, one obtains an approximation of incremental CO_2 emissions reduction costs, which are roughly $52 per ton of CO_2 for the first 17,000 tons and over $150 per ton for the last 10,000 tons of CO_2. Again, these results could be used by policy makers as guidelines and compared to society's willingness to reduce emissions in order to reach a plausible compromise.

Fig. 3 shows the annual generation mix for the different quota levels imposed. While the production of the CO_2 emission-free sources ("geo & renewables" and "hydro") remain basically unchanged, the dirty coal gets replaced steadily by the more expensive but cleaner bunker.

Figs. 4(a)-4(e) provide the monthly generation mix over the planning horizon for different quota levels. Most obvious are the changes in the electricity production with coal fired plants in the first stages when imposing high quotas. Observe that coal generation is replaced by bunker in the most cases. However, for the cases of relatively moderate emissions quotas of 3.90, 3.95 and 4.00 million tons of CO_2, the hydro resources are operated much more aggressively, leading to increases in the rationing risk (Table V). The hydro generation for the first stages increases (resulting in decreased water reservoir levels), leading to this higher risk. In contrast, for lower quota levels, less water is used in the first stages but instead the capacity of

TABLE I
THERMAL PLANTS CONSIDERED FOR THE GUATEMALA SYSTEM

Number of plants	1	3	1	18	3	1	1	3	10
Cumulative Capacity [MW]	24.0	120.4	41.4	729.8	91.5	132.4	13.0	58.0	227.0
Fuel Type	1	1	2	2	2	3	3	4	5
Cost [$/MWh]	129.9	132.0	61.6	67.1	68.7	41.2	45.9	2.7	1.0
CO_2 emissions [kg/MWh]	625.0	635.2	544.1	593.5	607.3	1001.0	1115.4	0	0

Fuel Type 1: Diesel, 2: Bunker, 3: Coal, 4: GEO, 5: Renewables

TABLE III
INSTALLED CAPACITY [MW] AND MONTHLY HYDRO INFLOWS [hm^3/month] FOR HYDRO-RESERVOIR AND RUN-OF-THE-RIVER PLANTS OF THE GUATEMALAN POWER SYSTEM

Station	Capacity	Inflow	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Annually
Chixoy	275.0	∅	51.46	39.32	32.01	29.63	38.71	128.28	127.66	146.61	207.82	166.14	78.01	64.98	1110.63
		min.	27.71	11.94	10.56	13.09	10.32	8.00	32.27	16.71	43.68	51.64	26.92	28.83	596.98
		max.	120.49	75.74	57.88	50.87	209.50	444.99	284.26	434.95	493.83	328.56	151.92	118.03	1901.10
		std.	14.45	12.96	11.46	8.52	21.75	71.39	52.25	83.49	108.23	61.39	26.73	17.35	301.74
Escalavos	14.0	∅	8.35	7.05	7.41	7.85	12.55	47.65	43.83	48.44	96.04	79.95	21.30	10.69	391.11
		min.	5.88	2.19	0.97	2.46	4.70	1.96	1.98	3.34	6.04	14.01	3.21	5.08	140.13
		max.	11.53	12.69	14.81	15.30	59.65	168.45	109.81	171.26	280.77	348.43	62.11	22.77	869.72
		std.	1.14	1.97	2.45	2.46	5.84	29.89	23.55	37.36	60.85	48.96	11.13	3.58	147.30
Aguacapa	80.0	∅	14.84	12.87	14.07	13.57	15.01	22.74	29.80	32.72	46.66	41.25	20.95	17.33	281.81
		min.	10.92	7.99	8.99	7.21	8.41	10.23	11.59	3.76	9.05	15.53	11.76	10.94	188.41
		max.	21.70	18.12	19.54	18.45	20.65	44.63	52.86	89.11	97.92	90.31	31.20	26.92	405.37
		std.	1.67	1.93	2.34	2.42	2.43	7.28	9.72	14.95	18.23	13.46	4.56	3.47	51.38
Jurun	60.0	∅	8.83	8.88	11.07	10.71	10.77	10.19	9.71	10.57	10.21	10.96	9.15	9.26	120.31
		min.	6.08	5.95	2.90	2.43	5.80	5.07	4.41	5.66	0.99	2.54	3.87	3.93	70.95
		max.	11.67	13.31	22.70	20.00	16.20	20.35	15.02	17.10	23.41	26.26	16.46	16.69	173.79
		std.	1.14	1.61	3.71	3.74	2.23	2.36	2.20	2.61	4.28	4.43	2.63	2.80	18.91
Renac	60.0	∅	51.74	33.37	27.87	22.02	22.06	62.87	84.94	86.59	102.63	119.46	86.62	72.18	772.35
		min.	12.94	13.46	11.11	10.26	6.94	20.76	20.53	21.40	37.67	48.53	22.95	19.38	539.68
		max.	108.74	83.57	68.94	39.86	76.18	134.64	176.49	182.67	264.88	211.88	188.99	221.77	1122.88
		std.	17.27	13.35	9.91	6.33	9.22	23.84	28.92	33.04	37.11	39.06	33.06	33.44	133.04
El Canada	40.0	∅	13.61	10.92	12.01	11.91	14.77	18.64	19.36	19.76	26.40	25.53	16.41	14.51	203.83
		min.	10.27	7.63	7.56	7.39	8.36	9.38	12.97	8.31	10.78	13.07	8.73	9.84	167.72
		max.	20.96	15.05	19.05	18.66	24.88	33.49	30.59	38.28	50.56	43.94	27.93	21.86	261.85
		std.	1.77	1.57	2.24	2.43	3.12	4.87	3.17	5.48	9.01	6.86	4.14	2.15	21.90
Las Vacas	38.0	∅	11.48	8.82	11.22	12.55	25.89	39.32	42.80	44.37	51.53	37.52	14.46	12.23	312.19
		min.	8.60	6.10	9.58	7.40	15.24	20.13	26.33	9.39	33.46	13.44	9.90	8.61	244.13
		max.	15.04	12.16	13.23	21.15	41.56	79.99	60.52	73.26	76.61	77.23	20.63	16.20	381.22
		std.	1.17	1.07	0.77	2.66	5.32	11.37	5.58	13.86	8.25	8.58	2.59	1.58	29.31
Matanzas	15.4	∅	5.01	3.80	3.25	2.80	3.11	6.25	8.08	7.95	9.92	9.19	5.87	5.61	70.84
		min.	2.62	1.65	1.22	0.72	0.98	1.71	3.02	2.27	2.63	2.52	1.65	2.18	38.28
		max.	9.84	8.31	6.48	5.95	8.46	15.06	17.87	16.25	27.06	20.02	15.01	10.70	105.57
		std.	1.30	1.27	0.99	1.22	1.33	2.47	2.98	3.01	4.56	3.26	2.46	1.76	12.01
Pasabien	12.0	∅	4.77	3.64	2.78	2.30	2.75	5.87	5.46	5.67	9.12	8.56	7.35	6.42	64.69
		min.	1.67	0.67	1.00	0.64	0.58	0.22	0.92	0.11	2.45	1.14	2.14	1.54	32.45
		max.	11.58	9.16	5.45	5.87	8.59	21.67	14.90	18.13	23.14	19.68	26.41	22.85	114.98
		std.	2.04	1.87	1.08	0.98	1.39	3.62	2.43	3.42	4.29	3.98	3.84	4.07	16.07
El Recreo	23.0	∅	13.61	10.92	12.01	11.91	14.77	18.64	19.36	19.76	26.40	25.53	16.41	14.51	203.83
		min.	10.27	7.63	7.56	7.39	8.36	9.38	12.97	8.31	10.78	13.07	8.73	9.84	167.72
		max.	20.96	15.05	19.05	18.66	24.88	33.49	30.59	38.28	50.56	43.94	27.93	21.86	261.85
		std.	1.77	1.57	2.24	2.43	3.12	4.87	3.17	5.48	9.01	6.86	4.14	2.15	21.90
Animas-Iz	10.0	∅	7.90	7.49	6.89	4.35	4.12	5.21	7.48	5.29	5.80	8.60	8.94	10.65	82.72
		min.	3.85	2.63	3.35	3.01	2.55	2.74	1.40	2.94	2.67	3.64	3.13	1.95	56.77
		max.	15.18	22.25	13.96	5.81	6.58	11.24	19.79	9.39	11.44	16.94	17.68	23.11	126.56
		std.	1.92	3.08	1.93	0.58	0.83	1.33	3.28	1.37	2.01	2.85	2.94	4.54	13.25

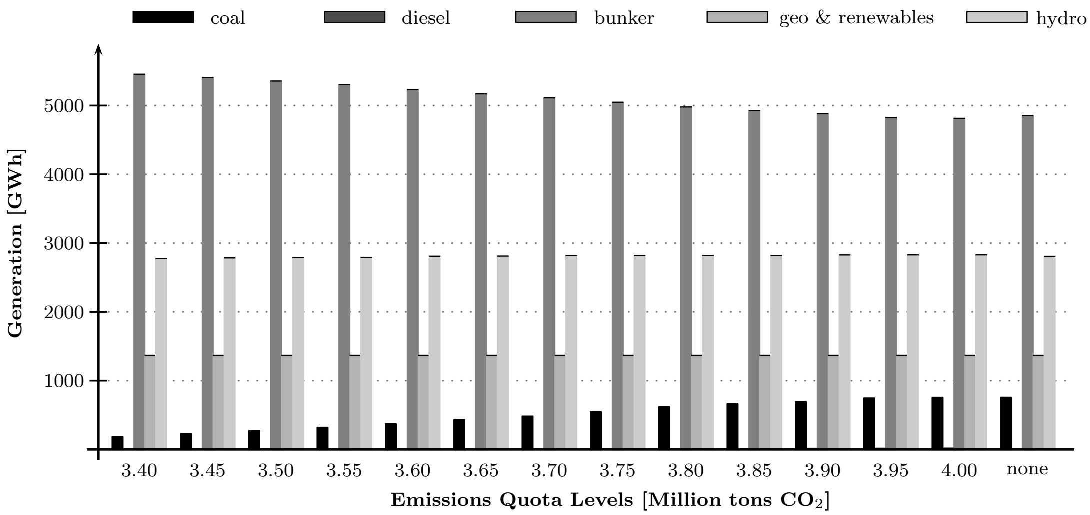


Fig. 3. Yearly generation mix for different quota levels

the bunker plants is used to produce electricity which, in later stages, could replace dirtier plants such as coal or diesel. As more of the inflow uncertainty unfolds, more coal can be used when it is expected that enough CO_2 emissions allowances are available.

The marginal CO_2 emissions prices and electricity prices are shown for the different quota levels in Figs. 5 and 6, respectively. The trend of decreasing prices is explained by the expiration of CO_2 emissions allowances at the end of the planning horizon of one year, *i.e.*, the CO_2 emissions rights have no future value beyond the planning horizon. The stochastic water inflow drives this trend further. While at the beginning, one might have to be very conservative with respect to CO_2 emissions for most (or even all) of the inflow scenarios, at the end of the horizon, the emissions quota might only affect a few of the scenarios. The water value at the end of the horizon is set to zero. Possible end effects of the hydro-thermal generation plan are very minor for this data set.

Stages 6, 7 and 8 (June, July and August) are the crucial stages for this data set. Out of the 100 inflow scenarios, several droughts occur during these stages, leading to a shortage in water and increasing the thermal production up to its capacity limits and with that, increasing the CO_2 emissions and the electricity production cost. Once these stages are past, the prices, both CO_2 allowance and electricity, drop. The generally relative high CO_2 emissions prices are explained by the energy system of Guatemala. To save one ton of CO_2 emissions generated by coal plants, one needs to replace roughly 2.2 MWh of coal fired plant production by bunker fuels, which leads in the cheapest case to an increase of $44.6 per MWh.

VI. Conclusions

In this paper, we discussed a CO_2 constrained hydro-thermal scheduling problem for the mid-term or long-term. To allow solving the problem via dynamic programming methods such as stochastic dynamic programming or stochastic dual dynamic programming, we propose a reservoir model for CO_2 emissions, respecting the stage decomposition framework of these methods. This reservoir model permits CO_2 emissions allowances to expire.

Computational results on the Guatemalan power system demonstrate the feasibility of the approach. The slope of the annual expected operational cost function for different quota levels provides the marginal *operational* CO_2 emissions reduction price. This yields useful insights for policy makers to establish socially acceptable CO_2 emissions quotas. Furthermore, the influence of different CO_2 emissions quotas on the electricity price are apparent.

The presented CO_2 emissions reservoir model can be applied to optimal expansion planning problems for hydro-dominated power systems. This can be achieved, for instance, by adapting the presented reservoir model of this article to the subproblems solved in the Benders Decomposition scheme as presented in [19]. This allows us to compute the marginal CO_2 emissions allowances price jointly for *investment* and *operations* planning.

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TABLE IV
MONTHLY ELECTRICITY DEMAND

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
GWh	801	738	828	771	844	805	840	847	806	850	820	838

TABLE V
COMPUTATIONAL RESULTS FOR DIFFERENT QUOTA LEVELS

Quota [million tons]	**3.40**	**3.45**	**3.50**	**3.55**	**3.60**	**3.65**	**3.70**	**3.75**	**3.80**	**3.85**	**3.90**	**3.95**	**4.00**	**NONE**
(Average) cost [million $]	370.1	367.0	364.5	362.4	359.5	357.6	355.8	353.8	351.6	351.1	350.8	349.3	349.3	348.4
Half-Width (95% confidence interval)	5.3	5.1	4.9	4.6	4.5	4.4	4.4	4.4	4.2	4.3	4.8	4.8	4.8	3.6
CO_2 emissions above quota [tons]	4914	3426	2425	1569	1109	634	427	274	162	57	23	24	0	-
# of scenarios above CO_2 emissions quota	46	36	28	25	19	18	14	10	10	4	1	1	0	-
Max. demand rationing [MWh %]	0	0	0	0	0	0.02	0.02	0.02	0.01	0.21	0.38	0.38	0.38	0
# of scenarios with demand rationing	0	0	0	0	0	1	1	1	1	3	6	6	7	0

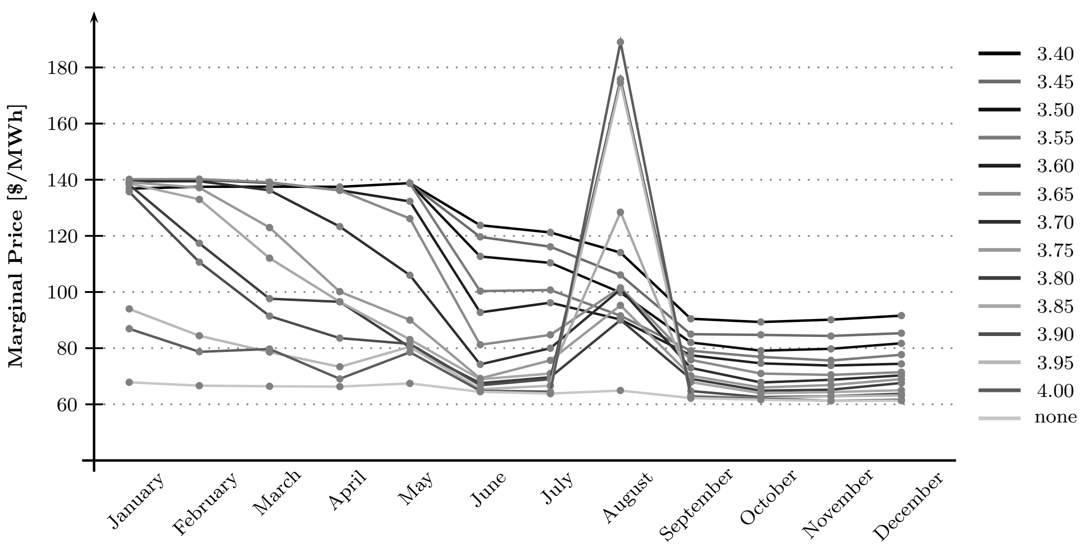


Fig. 5. Average electricity marginal prices for different quota levels

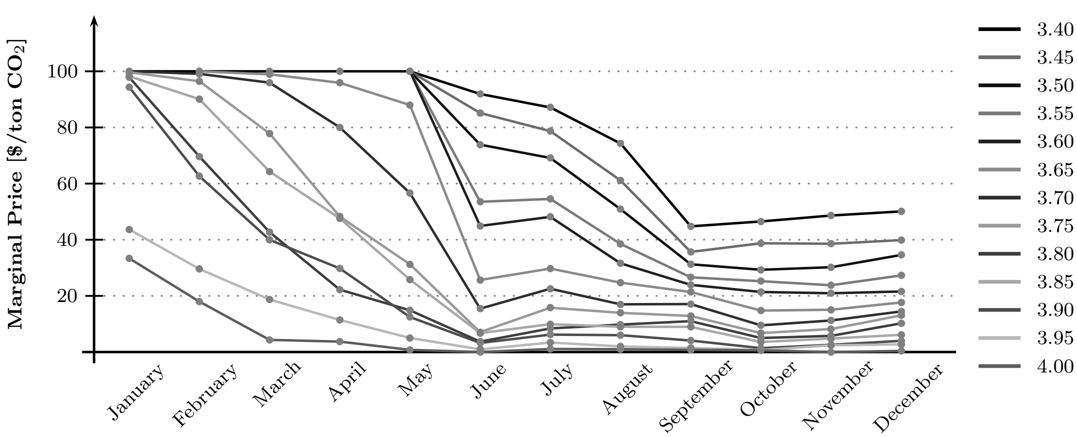
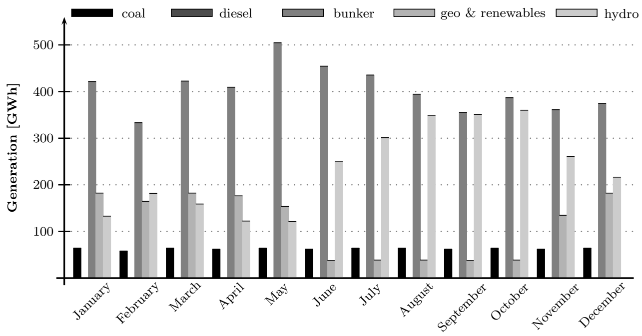
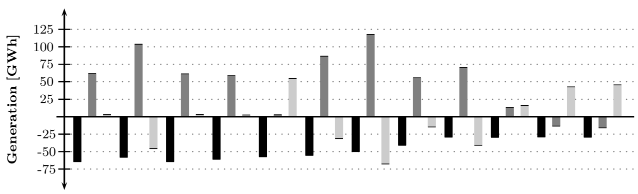


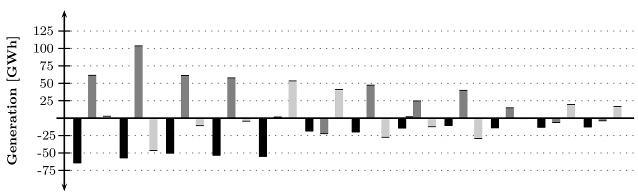
Fig. 6. Average CO_2 emissions allowance marginal prices for different quota levels



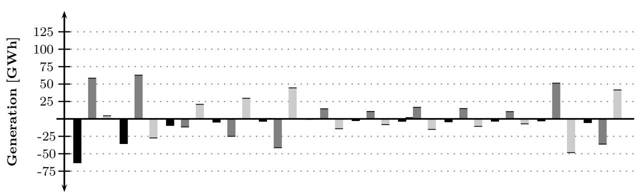
(a) Quota: None



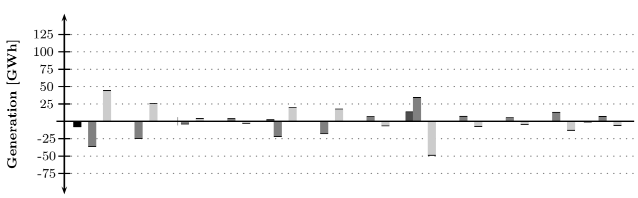
(b) Quota: 3.40 Million tons



(c) Quota: 3.60 Million tons



(d) Quota: 3.80 Million tons



(e) Quota: 3.95 Million tons

Fig. 4. Monthly dispatching decisions for the case of no quotas which is then used as the base case, showing the monthly difference in electricity generation

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