Generation Expansion Planning under Uncertainty with Emissions Quotas

Steffen Rebennack[a]

[a]*Division of Economics and Business, Colorado School of Mines, Golden, CO, USA*

Abstract

Generation expansion planning for hydro-thermal power systems aims to find optimal investment decisions among a set of possible power plant projects. For any given investment plan, expansion planning models must account for investment costs as well as expected operational costs. Additionally, when modeling hydro-thermal power systems, one must consider the inherent uncertainty in hydro inflows because variations in inflows can significantly impact operational decisions. To model the expansion planning problem for a hydro-thermal power producer, we propose a novel decomposition algorithm, based on Benders decomposition. The Benders master problem computes the investment decisions while the separation problems are emissions-constrained, least-cost, and stochastic hydro-thermal scheduling problems. The separation problems are solved using a standard stochastic dual dynamic programming (SDDP) algorithm. To demonstrate the effectiveness of the approach, we present a case study for Panama's power system. The computational results allow us to price carbon dioxide emissions reductions, aiding the evaluations of environmental policies.

Email address: `srebenna@mines.edu` (Steffen Rebennack)

Preprint submitted to Electric Power Systems Research *April 18, 2014*

Keywords: Optimal generation expansion planning, hydro-thermal power system, Benders decomposition, stochastic dual dynamic programming (SDDP), environmental policy, carbon dioxide emissions quotas

1. Introduction

In this paper, we examine generation expansion decisions for hydro-thermal power systems. We desire an investment portfolio that minimizes the investment cost plus the expected operational cost over the lifetime of the plants. We assume that inflow volatility has a significant impact on the operation of the power system. Consequently, we model the uncertainty in the inflows through stochastic programming techniques. In addition, we impose emissions quotas over a period of four years, mimicking the EU Emissions Trading System.

In order to solve the generation expansion problem, we develop a decomposition algorithm that is tailored to this problem. The algorithm makes investment decisions through Benders decomposition and the separation problems in the Benders decomposition scheme are emissions-constrained least-cost stochastic hydro-thermal scheduling problems. The emissions constraints are modeled via reservoir(s) [1]. The resulting large-scale stochastic linear programming problems (LPs) are solved using stochastic dual dynamic programming (SDDP) [2]. A unique feature of the proposed algorithm is that the separation problems can be solved using an off-the-shelf SDDP algorithm.

The algorithm most similar to ours can be found in [3]. In [3], the authors also employ Benders decomposition to solve an expansion planning problem where the separation problems are comprised of least-cost stochastic hydro-thermal scheduling problems. In contrast to our work, [3] does not use SDDP to solve the separation problems but an earlier methodology [4], also developed by the inventors of SDDP. Additionally, the model we present includes environmental constraints in terms of emissions quotas.

The body of literature on expansion planning problems is rich [5, 6]. Thus, we limit our summary to papers that study problems very similar to ours and/or utilize similar solution techniques. To solve a non-smooth expansion planning problem, a theoretically-sound bundle method is proposed in [7]. Uncertainties in pollution limits and electricity demand are represented using scenario trees; hydro plants are not included in their test example. Environmental considerations and their effects on electricity prices receive increasing interests [8, 9, 10].

In this paper, we develop a tool that can aid both system operators and policy makers. System operators are likely interested in optimal investment plans that take emissions quotas and inflow uncertainty into account. In contrast, policy makers may seek answers to the following questions: (i) What is a meaningful quota level? (ii) What are the economic and environmental implications/effects of a quota level? (iii) What are the operational consequences of a quota level? (iv) What are the consequences for the investment decisions of a quota level?

The developed model and solution algorithm has its application in the centralized planning environment, recognizing that planning is only of an indicative nature. In contrast, in a de-regulated electricity market environment, significant changes have to me made to the proposed methodology. Most notably, the least-cost hydro-thermal scheduling problems need to be replaced by a profit maximization model, *e.g.*, SDDP may be replaced by the hybrid SDP/SDDP algorithm [11, 12].

The paper contributes to the body of knowledge through (1) presenting a novel algorithm which solves a stochastic expansion planning problem using SDDP as a sub-routine taking quota levels into account, and (2) analyzing a case study for the Panama power system.

The remainder of the paper is organized as follows. Section 2 recounts the problem and outlines the solution algorithm; Section 3 presents the case study; and last, Section 4 provides concluding remarks.

2. Methodology

Our optimal power expansion planning problem can be formulated as

$$
\begin{aligned}
\text{(E)} \qquad z^* := \min \quad & c^1(\mathbf{x}^1) + c^2(\mathbf{x}^2) \\
\text{s.t.} \quad & f^1(\mathbf{x}^1) = \mathbf{b}^1 && (1) \\
& f^2(\mathbf{x}^1) + f^3(\mathbf{x}^2) = \mathbf{b}^2 && (2) \\
& f^4(\mathbf{x}^2) = \mathbf{b}^3 && (3) \\
& \mathbf{x}^1 \in \mathbb{N}^{n_1^1} \times \mathbb{R}_+^{n_1^2}, \quad \mathbf{x}^2 \in \mathbb{R}_+^{n_2} && (4)
\end{aligned}
$$

with investment decisions $\mathbf{x}^1$ and operational decisions $\mathbf{x}^2$. Functions $c^1(\cdot)$, $f^1(\cdot)$, $f^2(\cdot)$ are linear functions in the investment decisions; $c^2(\cdot)$, $f^3(\cdot)$, $f^4(\cdot)$ are linear functions in the operational decisions; $\mathbf{b}^1$, $\mathbf{b}^2$ and $\mathbf{b}^3$ are the right-hand-side vectors for constraints (1)-(3). Constraints (1) model the restrictions on the investment decisions, while (2) connect the investment decisions with the operational decisions. Constraints (3) model the operational constraints which are not directly affected by the investment decisions.

Optimization problem (E) is a two-stage optimization problem: the investment decisions are made "here-and-now" in a first stage, while the recourse decisions are the operational decisions that are made in a "wait-and-see" fashion. More precisely, the operational decisions are the optimal response to the investment decisions. Notice that the investment decisions are not scenario dependent, while they hedge against the stochasticity inherent in the operational problem [3]. As such, optimization problem (E) has the typical block diagonal structure.

After the investment decisions have been made, the remaining problem seeks to minimize costs while operating the hydro-thermal power system and adhering to system requirements. Thus, the second-stage problem is a least-cost stochastic hydro-thermal scheduling problem, which is itself a multi-stage optimization problem. Investment decisions are made over a period of several decades while stochastic hydro-thermal scheduling problems are typically solved with a weekly or monthly fidelity, capturing the inflow stochasticity. Thus, the investment and operational problems operate on different time scales. As a consequence, monolith formulation (E) is a very large-scale stochastic LP (SLP) or stochastic mixed-integer LP (MILP), depending on whether investment decisions $\mathbf{x}^1$ are continuous or discrete. Current state-of-the-art LP or MILP solvers cannot efficiently solve a monolith of this magnitude. As a solution, we propose Benders decomposition [13].

In Benders decomposition, the second-stage optimization problem, the so-called *separation problem* (S), is solved iteratively for trial investment decisions $\hat{\mathbf{x}}^1_k$ (index k is the iteration index, see Section 2.3)

$$
\begin{aligned}
\text{(S)} \qquad z(\hat{\mathbf{x}}_k^1) := \min \ & c^2(\mathbf{x}^2) \\
\text{s.t.} \ & f^3(\mathbf{x}^2) = \mathbf{b}_2 - f^2(\hat{\mathbf{x}}_k^1) && (\pi_k^2) && (5)\\
& f^4(\mathbf{x}^2) = \mathbf{b}^3 && (\pi_k^3) && (6)\\
& \mathbf{x}^2 \geq \mathbf{0},
\end{aligned}
$$

where π_k^2 and π_k^3 defines an optimal dual solution vector associated with constraints (5) and (6), respectively. SLP (S) is the operational problem which we discuss in detail in Section 2.2.

Because (S) is an LP and function $f^2(\cdot)$ is linear in $\mathbf{x}^1$, function $z(\cdot)$ is piecewise linear and convex in investment decisions $\mathbf{x}^1$. Thus, $z(\cdot)$ can be linearly approximated for any investment decision $\mathbf{x}^1$, which satisfies (1) and (4), with the following so-called Benders *optimality cut*

$$
z(\mathbf{x}^1) \geq \pi_k^2 \left(\mathbf{b}^2 - f^2(\mathbf{x}^1)\right) + \pi_k^3 \mathbf{b}^3. \tag{7}
$$

We assume that LP (S) is feasible for any trial investment decision $\hat{\mathbf{x}}^1$ satisfying (1) and (4), *i.e.*, we assume that (S) has relative complete recourse. This implies that no *feasibility cuts* are required.

The *approximate power expansion planning problem* $(\widetilde{\mathrm{E}})$ uses the Benders cuts to approximate function $z(\cdot)$ as

$$
\begin{aligned}
(\widetilde{\mathrm{E}}) \qquad \underline{z} := \min \;\; & c(\mathbf{x}^1) + \eta \\
\text{s.t.} \;\; & \eta \geq \pi_\kappa^2 \left(\mathbf{b}^2 - f^2(\mathbf{x}^1)\right) + \pi_\kappa^3 \mathbf{b}^3 \quad \forall \kappa \in \mathbb{K} \\
& f^1(\mathbf{x}^1) = \mathbf{b}^1 \\
& \mathbf{x}^1 \in \mathbb{N}^{n_1^1} \times \mathbb{R}_+^{n_1^2},
\end{aligned}
$$

where set $\mathbb{K}$ collects all the computed cuts so far; η is an unrestricted decision variable of dimension 1. Any optimal solution to $(\widetilde{\mathrm{E}})$ serves as a new trial solution for the separation problem. $(\widetilde{\mathrm{E}})$ is called the *master problem.*

Because the Benders cuts underestimate the expected operational cost, $\underline{z}$ is a lower bound on z^*. An upper bound, $\bar{z}$, for $\hat{\mathbf{x}}_k^1$ on z^* is given by $c(\hat{\mathbf{x}}_k^1) + z(\hat{\mathbf{x}}_k^1)$. The Benders algorithm converges, if $\frac{\bar{z}-\underline{z}}{\bar{z}} \leq \varepsilon$, for optimality tolerance $\varepsilon > 0$.

2.1. Investment Problem

Let there be P potential projects $p \in \mathbb{P} := \{1, \dots, P\}$ to expand generation capacity. The decision is then to either invest or not invest in a particular project p. The timing of the investment is crucial. Let there be T stages, $t \in \mathbb{T} := \{1, \dots, T\}$, where each stage represents a monthly time step. We allow to realize project $p \in \mathbb{P}$ at stages $t \in \mathbb{T}_p$, $\mathbb{T}_p \subset \mathbb{T}$, modeled via the binary decision variables

$$
x_{pt}^1 = \begin{cases} 1, & \text{if project } p \text{ is realized at stage } t \\ 0, & \text{otherwise} \end{cases},
$$

representing vector $\mathbf{x}^1$ in (E); $n_1^1 = P$ and $n_1^2 = 0$. If $x_{pt}^1 = 1$, then we assume that the power plant(s) associated with project p become available at stage t and remain available for all future stages $\tau \geq t$, $\tau \in \mathbb{T}$.

The investment cost, *i.e.*, first stage decision cost, are given by

$$c^1(\mathbf{x}^1) := \sum_{p \in \mathbb{P}} \sum_{t \in \mathbb{T}_p} c_{pt}^{\mathrm{I}} x_{pt}^1,$$

with discounted and adjusted investment costs, c_{pt}^{I}, for project $p \in \mathbb{P}$ realized at stage $t \in \mathbb{T}_p$.

The investment decisions are subject to the following constraints:

$$\sum_{t \in \mathbb{T}_p} x_{pt}^1 \leq 1 \tag{8}$$

$$\sum_{t \in \mathbb{T}_p} x_{pt}^1 = 1 \qquad p \in \mathbb{P}^{\mathrm{M}} \tag{9}$$

$$\sum_{\varrho \in \mathbb{P}_p^{\mathrm{C}}} \sum_{t \in \mathbb{T}_\varrho} x_{t\varrho}^1 \leq 1 \qquad p \in \mathbb{P} \tag{10}$$

$$\sum_{t \in \mathbb{T}_p} x_{pt}^1 = \sum_{t \in \mathbb{T}_\varrho} x_{t\varrho}^1 \qquad p \in \mathbb{P},\ \varrho \in \mathbb{P}_p^{\mathrm{A}}. \tag{11}$$

Constraint (8) ensures that each project can be realized at most once among the stages in $\mathbb{T}_p$; constraints (9) force the investment into mandatory projects $p \in \mathbb{P}^{\mathrm{M}}$, *i.e.*, projects $p \in \mathbb{P}^{\mathrm{M}}$ must be realized at exactly one of the stages in $\mathbb{T}_p$; constraints (10) forbid investments into projects $\varrho \in \mathbb{P}_p^{\mathrm{C}}$ conflicting with project p, *i.e.*, projects $\varrho \in \mathbb{P}_p^{\mathrm{C}}$ and p cannot both be realized over the planning horizon; constraints (11) model associated or interdependent projects, *i.e.*, projects p and $\varrho \in \mathbb{P}_p^{\mathrm{A}}$ can only be realized together, *i.e.*, you can invest in project p, if and only if you invest in project $\varrho \in \mathbb{P}_p^{\mathrm{A}}$. As such, constraints

(8)-(11) represent (1).

2.2. Operational Problem

Let $\mathbb{I}$ denote the set of hydro plants; let $\mathbb{I}_p$ be the set of all hydro plants/reservoirs associated with project p; let $\mathbb{J}$ be the set of all thermal plants; let $\mathbb{J}_p$ denote the set of thermal plants that are associated with project p. Both sets $\mathbb{I}$ and $\mathbb{J}$ contain the reservoirs and plants which are not yet built, *i.e.*, $\mathbb{I}_p \subset \mathbb{I}$ and $\mathbb{J}_p \subset \mathbb{J}$ for all projects p.

In the following discussion, we assume that $t \in \mathbb{T}$ and that the water inflows A_{it} into reservoir $i \in \mathbb{I}$ at stage t are given, *e.g.*, as one realization of the stochastic water inflow scenarios, see Section 2.3.

We start with the constraints affecting the hydro reservoirs. Each hydro plant $i \in \mathbb{I}$ is subject to the water balance equations

$$v_{i,t+1} = v_{it} - u_{it} - s_{it} + \sum_{h \in \mathbb{U}_i} (u_{ht} + s_{ht}) + A_{it}. \tag{12}$$

The water balance equations (12) for reservoir $i \in \mathbb{I}$ state that the reservoir level at the end of stage t (non-negative decision variable $v_{i,t+1}$ [hm^3]) equals the reservoir level at the beginning of stage t minus the water releases due to generation (non-negative decision variable u_{it} [hm^3]) and spillage (non-negative decision variable s_{it} [hm^3]) plus the upstream activity (set $\mathbb{U}_i$ is made up of the hydro plants that are immediately upstream of plant i) plus the water inflow.

The hydro generation is subject to the constraints

$$\underline{U}_{it} \le u_{it} \le \overline{U}_{it} \qquad i \in \mathbb{I} \tag{13}$$

$$\underline{U}_{it} \sum_{\tau \in \mathbb{T}_p, \tau \le t} x^1_{p\tau} \le u_{it} \le \overline{U}_{it} \sum_{\tau \in \mathbb{T}_p, \tau \le t} x^1_{p\tau} \qquad i \in \mathbb{I}_p,\ p \in \mathbb{P}, \tag{14}$$

with minimum release level $\underline{U}_{it}$ [hm^3] and maximum release level $\overline{U}_{it}$ [hm^3].

The hydro reservoir levels are subject to lower and upper limits, $\underline{V}_{i,t+1}$ [hm^3] and $\overline{V}_{i,t+1}$ [hm^3], enforced by constraints

$$\underline{V}_{i,t+1} \le v_{i,t+1} \le \overline{V}_{i,t+1} \qquad i \in \mathbb{I} \tag{15}$$

$$\underline{V}_{i,t+1} \sum_{\tau \in \mathbb{T}_p, \tau \le t+1} x^1_{p\tau} \le v_{i,t+1} \le \overline{V}_{i,t+1} \sum_{\tau \in \mathbb{T}_p, \tau \le t+1} x^1_{p\tau} \qquad i \in \mathbb{I}_p,\ p \in \mathbb{P}. \tag{16}$$

To appropriately quantify the value of the hydro projects, it is crucial that the water can flow through all hydro-plants, including the ones which are not yet built. As a result, the spillage variables s_{it} must not be bounded by the investment decisions x^1_{pt}. The dual decision variables that correspond to constraints (14) and (16) decide on the value of the hydro projects, see Section 2.3.

Similar to the generation from the hydro plants, the thermal generation (non-negative decision variable g_{jt} [MWh]) has minimum generation $\underline{G}_{jt}$ [MWh] and maximum generation $\overline{G}_{jt}$ [MWh], *i.e.*,

$$\underline{G}_{jt} \le g_{jt} \le \overline{G}_{jt} \qquad j \in \mathbb{J} \tag{17}$$

$$\underline{G}_{jt} \sum_{\tau \in \mathbb{T}_p, \tau \le t} x^1_{p\tau} \le g_{jt} \le \overline{G}_{jt} \sum_{\tau \in \mathbb{T}_p, \tau \le t} x^1_{p\tau} \qquad j \in \mathbb{J}_p,\ p \in \mathbb{P}. \tag{18}$$

The CO_2 emissions of the thermal generators $j \in \mathbb{J}$ are given by the emissions coefficient B_j [tons CO_2/MWh]. We incorporate emissions via a "reservoir" model, instead of having a single constraint spanning multiple stages, *e.g.*, 4 years or 36 stages. This emissions "reservoir" takes the role of a hydro reservoir, where it "rains" emissions quotas at pre-defined stages. Nevertheless, the emissions "reservoir" is a modeling construct and as such disconnected from the water balance equations (12). The reservoir model is chosen in order to respect the block-diagonal structure of the operational problem to allow for its efficient solution [14, 1]. It is given by

$$e_{t+1} = e_t - \sum_{j \in \mathbb{J}} B_j g_{jt} \qquad t \in \mathbb{T} \setminus \mathbb{T}^{\mathrm{E}} \tag{19}$$

$$e_{t+1} = E_t^{\mathrm{CO_2}} - \sum_{j \in \mathbb{J}} B_j g_{jt} \qquad t \in \mathbb{T}^{\mathrm{E}}. \tag{20}$$

Set $\mathbb{T}^{\mathrm{E}} \subset \mathbb{T}$ contains the stages when new CO_2 emissions allowances become available; non-negative decision variable e_{t+1} [tons CO_2] represent the emissions allowances left at the end of stage t; and $E_t^{\mathrm{CO_2}}$ [tons CO_2] are the emissions allowances which become available at stages $t \in \mathbb{T}^{\mathrm{E}}$. Constraints (19)-(20) represent the balance equations for the CO_2 emissions allowances, *i.e.*, the number of allowances left at the end of stage t equals the number of allowances available at the beginning of stage t minus the allowances "consumed" by thermal generation. In this model, the previous emissions allowances expire, *cf.* constraints (20). Generation from renewables, including hydro, are assumed to have zero emissions.

The electricity demand D_t [MWh] is to be met by thermal and hydro generation, or mathematically,

$$\sum_{j \in \mathbb{J}} g_{jt} + \sum_{i \in \mathbb{I}} \rho_i u_{it} = D_t, \tag{21}$$

with hydro generation efficiency coefficient ρ_i [MWh/hm^3]. Though the hydro production is a non-linear function in the water reservoir level, we follow the common assumption in mid-term to long-term scheduling problems to use a linear approximation; *cf.* Section 2.4.

Finally, the expected operational cost minimizes the sum of thermal generation plus penalty terms for electricity rationing, *e.g.*, violating (21), and emissions overspending, *e.g.*, violating (19) and (20).

Because we assume that the inflows A_{it} are non-negative and that the initial water reservoir levels satisfy the lower and upper reservoir level constraints, the operational problem is always feasible, regardless of the inflow scenario and the investment decisions taken. This is ensured by making the emissions constraints (19) and (20) and the demand constraints (21) soft-constraints via the introduction of the aforementioned penalty terms.

The non-negative decision variables $v_{i,t+1}$, u_{it}, s_{it}, g_{jt} and e_{t+1} as well as the penalty terms (not explicitly stated above) represent the operational decision vector $\mathbf{x}^2$ in (E). Constraints (14), (16) and (18) are the coupling constraints (2); constraints (12), (13), (15), (17), (19) - (21) model the operational constraints (3).

2.3. Solving the Operational Problem using SDDP and Computing the Benders Cuts for the Master Problem

For a binary trial decision vector $\hat{\mathbf{x}}_k^1$, the operational problem in constraints (12)-(21) is a least-cost hydro-thermal scheduling problem with uncertain inflows. Such problems are solved using dynamic programming approaches. For large cascade systems, SDDP is the current state-of-the-art. The SDDP method is a nested Benders decomposition approach which samples the inflow scenarios from the scenario tree that models the uncertainty. SDDP consists of a *backward recursion*, essentially computing Benders cuts and a lower bound (for minimization problems), and a *forward simulation*, yielding an (approximate) upper bound as well as an operations policy. The SDDP algorithm converges once the (approximate) upper and lower bound are within a confidence interval [2, 15, 16]. The main advantage of the SDDP method compared to stochastic dynamic programming related methods is that, for practical purposes, it breaks the curses-of-dimensionality inherent from the "explosion" of the state or scenario space in the reservoir discretization, scenarios, and time stages.

Assume the operational problem has been solved for trial decision vector $\hat{\mathbf{x}}_k^1$ with components $\hat{x}_{kpt}^1$. Next, we desire to compute the Benders cut (7). Also assume that the forward simulation in SDDP consists of $m \in \mathbb{M}$ inflow scenarios, occurring with probability p_m; naturally, $\sum_{m \in \mathbb{M}} p_m = 1$. For iteration (cut) index k and forward simulation pass $m \in \mathbb{M}$, let $\pi_{ikmpt}^{\underline{\mathrm{U}}}$, $\pi_{ikmpt}^{\overline{\mathrm{U}}}$, $\pi_{ikmpt}^{\underline{\mathrm{V}}}$, $\pi_{ikmpt}^{\overline{\mathrm{V}}}$, $\pi_{jkmpt}^{\underline{\mathrm{G}}}$, and $\pi_{jkmpt}^{\overline{\mathrm{G}}}$ be an optimal dual solution corresponding to constraints (14), (16) and (18).

The variable Benders cut component, $-\pi_k^2 f^2(\mathbf{x}^1)$, in (7) is then given by

$$\sum_{p\in\mathbb{P},t\in\mathbb{T}_p} \pi_{kpt}^{\mathrm{MP}} x_{pt}^1$$

with

$$\pi_{kpt}^{\mathrm{MP}} = \sum_{\tau\geq t}\sum_{m\in\mathbb{M}} p_m \Bigg(\sum_{i\in\mathbb{I}_p} \Big(\pi_{ikmp\tau}^{\underline{\mathrm{U}}} \underline{U}_{i\tau} + \pi_{ikmp\tau}^{\overline{\mathrm{U}}} \overline{U}_{i\tau} + \pi_{ikmp\tau}^{\underline{\mathrm{V}}} \underline{V}_{i,\tau+1} + \pi_{ikmp\tau}^{\overline{\mathrm{V}}} \overline{V}_{i,\tau+1} \Big) + \sum_{j\in\mathbb{J}_p} \Big(\pi_{jkmp\tau}^{\underline{\mathrm{G}}} \underline{G}_{j\tau} + \pi_{jkmp\tau}^{\overline{\mathrm{G}}} \overline{G}_{j\tau} \Big) \Bigg)$$

for $p \in \mathbb{P}$ and $t \in \mathbb{T}_p$. The cut constant, $\pi_k^2 \mathbf{b}^2 + \pi_k^3 \mathbf{b}^3$, is computed through

$$z(\hat{\mathbf{x}}_k^1) - \sum_{p\in\mathbb{P},t\in\mathbb{T}_p} \pi_{kpt}^{\mathrm{MP}} \hat{x}_{kpt}^1,$$

where $z(\hat{\mathbf{x}}_k^1)$ is the expected operational cost associated with the computed generation policy.

The resulting Benders decomposition algorithm is illustrated in Figure 1. The separation problems (S) are solved using SDDP, while the investment decisions are found in the master problem.

2.4. Model Enhancements

There are several ways to further develop our optimal expansion model, including the following extensions of the operational problem:

- adding power transmission restrictions or energy transfer limits between different areas of the power system [17]; because the AC distribution system is governed by non-convex relationships a DC or pipeline model might be a good first approximation [18, 19],

- incorporating gas network considerations, which follow non-linear physical relationships [20],
- modeling non-linear head effects [21, 22],
- reflecting non-linear, typically quadratic, thermal generation cost [23, 24] or discrete power plant dispatching [25, 26],
- integrating financial risk measures and reliability constraints on electricity supply [27, 28],
- considering reliability associated with power generation and transmission [29],
- representing constraints on fossil fuels, such as "min-take" or "take-or-pay" contracts or fuel switching of thermal plants,
- including different threshold values for the cost/penality of electricity demand rationing, and
- modeling additional uncertainties, for instance, in the electricity demand and/or the fuel prices via a scenario tree approach [30, 31].

However, it is vital to maintain the problem structure in (E), in order to allow for the developed solution method to work. This seems like a contradiction, because the non-convex relationships imposed by the first four to six mentioned extensions destroy this structure. However, recent convexification approaches, based on Lagrangian relaxation of the water balance equations, allow for their approximation and incorporation into the Benders decomposition framework [32, 33, 34].

Similarly, the investment problem can be extended to incorporate, for instance, transmission expansion [35, 36], retirement planning [37], investment ceiling in form of budget constraint(s), big investment projects which are not realized by a single stage but become available over time, and earliest date of commissioning instead of pre-defined stages for the investment realization.

3. Panama Case Study

We use Panama's hydro-thermal power system for our computational tests. The system is comprised of four hydro reservoirs and a run-of-the-river plant with a combined water storage capacity of 3,353 hm^3, see Figure 2. There is a total installed capacity of 842 MW of thermal generators (diesel, coal and bunker). We use a 5% discount rate and 5% annual demand growth rate over the planning horizon of 16 years. The stochastic water inflows are modeled via a period autoregressive model of lag 1 [38, 39] with a monthly resolution, based on historical inflow data. We use ten forward inflow series, *i.e.*, $|\mathbb{M}| = 10$, and 25 backward openings within the SDDP algorithm. Thus, the operational problem spans 192 stages. We consider the investment into 17 expansion projects with three possible investment periods at the beginning of years 1, 5 and 10, *i.e.*, $\mathbb{T}_p = \{1, 49, 109\}$. There are six fossil-fuel projects (coal, bunker, diesel) and five renewable energy projects other than hydro (wind, geo), see Table 1 (column labeled "O&M cost" reports on the operation and maintenance costs of the plants). Both wind and geo are modelled as non-dispatchable thermal plants with equal minimum and maximum generation. The six hydro investment project choices are listed in Table 2. The investment cost for thermal and hydro projects are discounted and adjusted appropriately to reflect the finite planning horizon. We set the fine for not meeting the emissions quotas to \$100 per ton CO_2 and penalize demand rationing by 8 times the maximal generation cost, *i.e.*, a penalty of \$3,168.56 per MWh.

We implement both the SDDP algorithm for the operational problem (S) and the Benders decomposition algorithm for the investment problem, with Xpress-Mosel (version 3.4.3). We solve the resulting LPs and MILPs with the Xpress-Optimizer (version 25.01.05) and execute the computations on an Intel(R) Core(TM) i7 @ 2.93 Ghz with 12 GB RAM and 64-bit Windows 7 Professional, using a single core. We stop the SDDP algorithm (for a particular trial investment decision vector) if a 95% confidence interval is reached or three backward-forward iterations are executed and we stop the Benders decomposition algorithm (for the investment problem) when a relative gap of 0.5% is reached, *i.e.*, $\varepsilon = 0.005$. The computational results are summarized in Table 3.

The LPs (without Benders cuts) have 141 decision variables and 286 constraints and the MILPs (without Benders cuts) have 52 decision variables (51 are binary) and 21 constraints; trivial sizes for current LP and MILP solvers. However, the number of LPs to be solved is quite large, *cf.* Table 3. The MILPs of the master problem ($\widetilde{\mathrm{E}}$) tend to be more computational challenging with increasing number of Benders cuts added. However, all master problems solve very quickly compared to the operational problems, *cf.* Table 3. Thus, improvements in solving the operational problems directly translate to improvements of the proposed expansion algorithm. In particular, commercially available SDDP algorithms are several orders of magnitude faster than the presented SDDP implementation. Consequently, expansion planning problems for much larger hydro-power systems can be solved within a similar time frame as reported in Table 3.

The results for the Panama case study are illustrated in Figures 3 and 4. Figure 3 shows the investment and operational costs associated with different emissions quota levels, while Figure 4 shows the associated investment decisions for each of the three possible investment periods.

The average annual CO_2 emissions amount to 3.83 million tons in the base case with no emissions quotas. Introducing emissions quotas of 19 million tons per four years, that is an annual quota of 4.75 million tons CO_2, already has a significant impact on the annual emissions. They drop to 3.36 million tons, on average, that is an average emissions reduction of 12.3%. The investment decisions swap the investment of coal in year 5 with the investment into hydro in year 10. The total cost increases only slightly, which results in an annual price of \$10.2 per ton CO_2. Note that the emissions reduction results from the hedging against the inflow scenarios with high CO_2 emissions.

Between 19 and 13 million tons CO_2, the total cost and the emissions savings grow approximately linearly. The total cost of one tone CO_2 reduction per year, on average, amounts to approximately \$28.5, comprised of changes in the investment and operational cost. These marginal CO_2 emissions costs include the fines paid for not meeting the set emissions quotas. The observed emissions savings (on average) for a quota level of 13 million tons CO_2 leads to approximately 23% emissions savings compared to the case of a quota of 19 million tons CO_2 and to approximately 32% reduction compared to the base case. The cost per emissions saving can be lowered, for instance, by adding additional projects to the investment portfolio or by lowering the fine(s).

We further observe that it takes a quota level of 16 million tons CO_2 per four years in order for investments into wind power to become profitable, in the final period. The quota of 16 million tons CO_2 also marks the first significant change in the investment decision. All other quota levels (19, 18 and 17 million tons CO_2) are dealt with through adjusting only the operation's policy. We see the first investments into wind at the second investment period at the very aggressive quota level of 7 million tons CO_2. Further, coal is very attractive up to a quota level of 6 million tons.

Until a quota of 10 million tons of CO_2, the operational cost slightly decrease. This is due to the zero-operational cost of the renewable generators. These cost savings are offset by the emissions fines for stricter quota levels, leading to a rapid increase in the operational costs.

4. Conclusions

In this paper, we present a decomposition algorithm which can solve a stochastic generation expansion planning problem for a hydro-thermal power system. Specifically, we propose an algorithm that models uncertainty in the inflows and seeks optimal operational decisions. The resulting hydro-thermal scheduling problems are solved using stochastic dual dynamic programming. The developed tool allows policy makers to set quota levels and to quantify the emissions savings and operational as well as investment cost changes associated with their imposed quota levels. For the Panama case study, we show that investments in wind resources become attractive at moderate emissions quota levels while very aggressive emissions restrictions are necessary to eliminate coal from the optimal investment portfolio. We further demonstrate that the existence of an, apparently loose, quota level leads to considerable emissions reductions at moderate cost.

Acknowledgements

The author thanks Mario V.F. Pereira (PSR, Brazil) for his inspiring discussions in an early stage of this research and Bruno Flach (IBM research, Brazil) for his help with the case study.

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Type	**Capacity** [MW]	**Mean capacity factor**	**Inv. cost** [Million $]	**Years**	**O&M cost** [$/MWh]	**Emissions** [tons CO_2/MWh]
wind	50	34%	230.00	15	10.00	0.000
wind	75	34%	345.00	15	10.00	0.000
wind	150	34%	690.00	15	10.00	0.000
GEO	40	60%	310.00	15	25.00	0.000
GEO	80	60%	620.00	15	25.00	0.000
coal	150	70%	273.70	20	56.97	1.459
coal	250	75%	456.44	20	56.41	1.444
coal	500	80%	775.95	20	43.67	1.115
bunker	100	70%	125.93	20	68.70	0.559
diesel	100	75%	81.57	20	152.89	0.736
diesel	250	80%	265.62	20	114.68	0.552

Table 1: Thermal expansion projects

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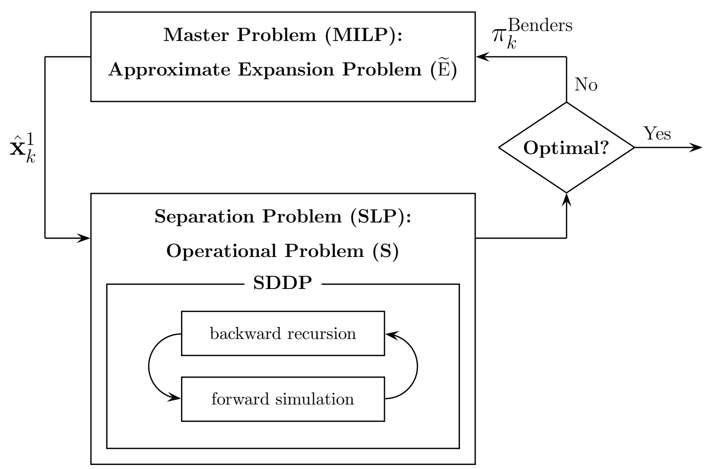


Figure 1: Solution algorithm for the optimal power expansion planning problem combining Benders decomposition with SDDP

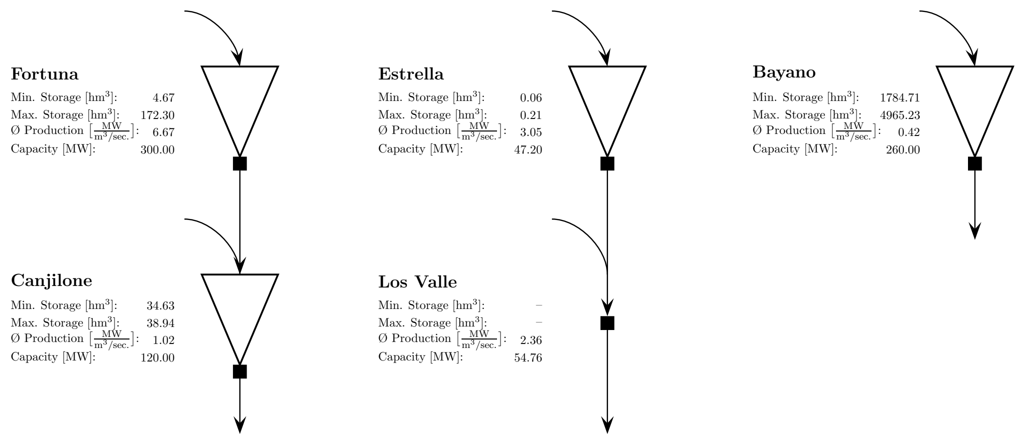


Figure 2: Cascade system of Panama's installed hydro-plants [30]

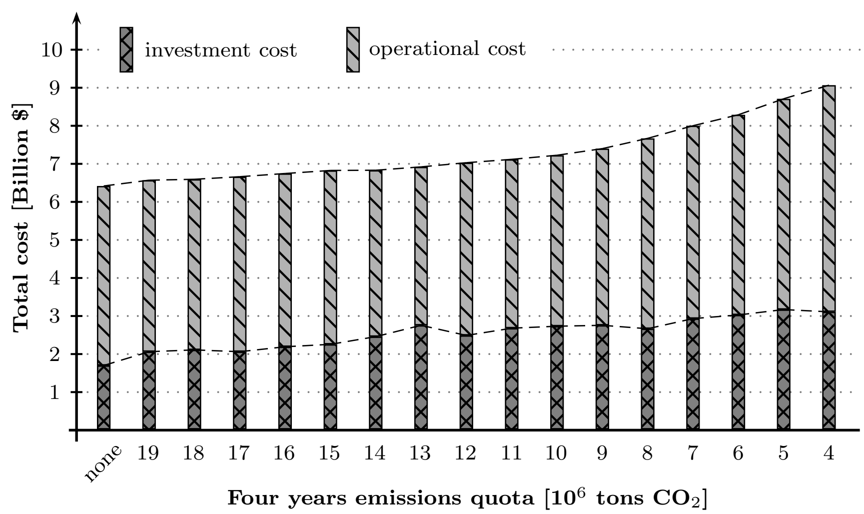
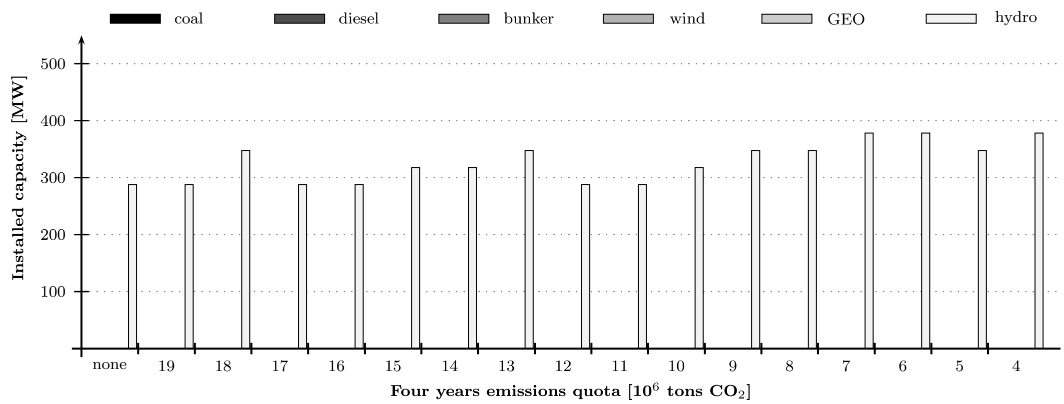


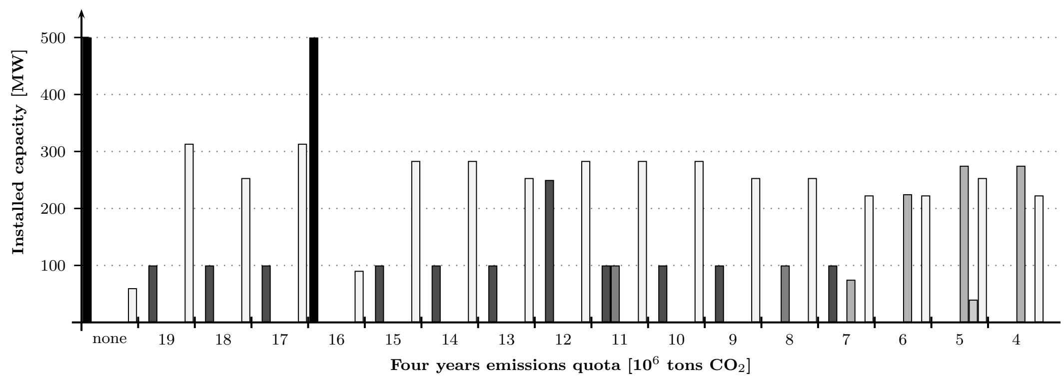
Figure 3: Total cost for different quota levels

Name	Type	Capacity [MW]	Inv. cost [Million $]	Years
PA_S. María	reservoir	30.50	255.50	30
PA_Chan I	reservoir	222.99	2292.50	30
PA_Chan II	reservoir	132.06	1137.50	30
PA_Chan220	run-of-the-river	126.35	519.00	30
PA_Bonyic	run-of-the-river	29.98	117.00	30
PA_El Alto	run-of-the-river	60.18	234.00	30

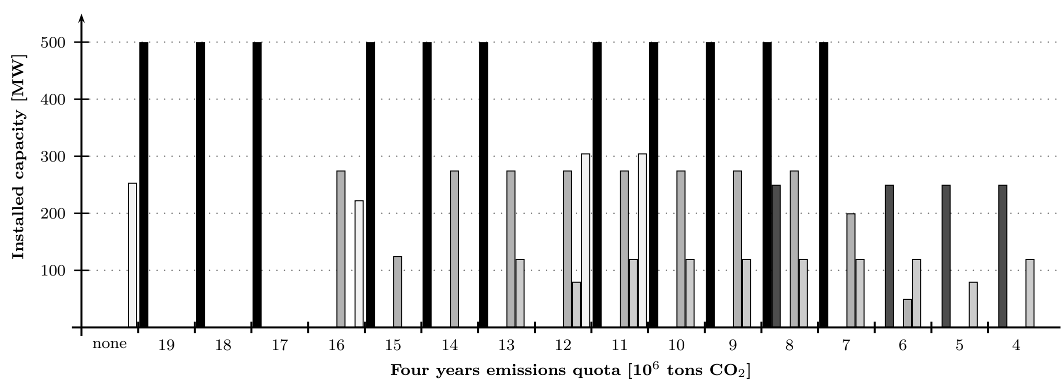
Table 2: Hydro expansion projects



(a) Year 1



(b) Year 5



(c) Year 10

Figure 4: Optimal expansion investment decisions for different four year quota levels

Four years quota [10^6 tons CO_2]	**LB** [10^9 \$]	**UB** [10^9 \$]	**# Benders iterations**	**# LPs solved**	**CPU time** ($\widetilde{\mathrm{E}}$) [sec.]	**total** [sec.]
none	6.4636	6.4948	42	2,086,560	15.6	6,693.0
19	6.5129	6.5448	38	5,663,520	14.5	19,317.2
18	6.5728	6.5983	45	6,706,800	16.7	22,734.1
17	6.6469	6.6798	47	7,004,880	18.4	23,807.4
16	6.7176	6.7465	60	8,942,400	27.3	30,680.0
15	6.7910	6.8247	63	9,389,520	25.9	32,149.2
14	6.8002	6.8338	44	6,557,760	15.9	22,432.0
13	6.8870	6.9214	46	6,855,840	17.1	23,305.9
12	6.9971	7.0305	51	7,601,040	19.9	25,956.4
11	7.0796	7.1096	48	7,153,920	19.0	24,496.7
10	7.1946	7.2265	36	5,365,440	12.0	19,326.7
9	7.3917	7.4007	27	4,024,080	7.5	13,592.1
8	7.6385	7.6715	30	4,471,200	8.7	15,523.2
7	7.9648	8.0022	32	4,769,280	9.6	17,427.1
6	8.3407	8.3826	29	4,322,160	8.0	14,670.2
5	8.6697	8.7103	22	3,278,880	5.3	11,757.2
4	9.0404	9.0859	25	3,726,000	6.5	12,608.7

Table 3: Computational results